A SIGNAL SUBSPACE APPROACH FOR SINGLE-TRIAL EVOKED POTENTIAL ESTIMATION

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Abstract: Evoked potentials (EP) are electro-physiological signals generated by the central nervous system in response to stimulation. Subspace methods in EP analysis commonly relate to the application of the singular value decomposition (SVD) to reveal the principal features of the signals. In single-channel EP analysis, the measurement matrix is constructed based on an ensemble of single-trial EPs, obtained from repeated presentation of stimuli. In this paper, a new subspace based method for EP enhancement is presented. The method exploits ensemble as well as within single sweeps waveform information. The proposed single-trial estimation method is applied to auditory EPs and outperforms the ensemble approach.

1. Introduction

The electroencephalogram (EEG) is the recording of the potential differences between electrode locations placed on the scalp. EEG records can be classified into two categories: the spontaneous or background EEG and EPs. The background EEG consists of voltage fluctuations, which can be classified according to their frequency content, for example, alpha activity (8-12 Hz). An EP is usually considered to be a wave or complex elicited by and time-locked to a physiological or non-physiological stimulation or event. However, EPs are buried into background brain activity and non neural activity, such as muscle noise. The spectrum of the background EEG overlaps with that of the EP and therefore, frequency selective filters cannot be used.

The simplest way to investigate EPs is to use ensemble averages of time-locked EEG epochs obtained by repeated stimulation. Since the background EEG is not assumed to have a temporal relationship to the synchronization event, averaging of a large number of EEG epochs will result in its attenuation. The activity that does have a temporal relation to the event, i.e. the EP, will then be enhanced and thus become more visible for analysis. However, it is well known that this signal enhancement leads to loss of information related to trial-to-trial variability.

Several methods have been proposed for EP estimation and de-noising. The performance of every single-trial estimation method depends on the prior information used and the statistical properties of the EP signals. The subspace method implemented using SVD has been used for enhancing non-averaged data for single-channel EEG analysis. In this approach, the space of the observed data is decomposed into signal and noise subspaces. Since the signal and noise subspaces are orthogonal, projecting the observed signal onto the signal subspace leads to the reduction of noise.

However, SVD based on the ensemble EP measurements can efficiently decompose the signal space if the signal-to-noise ratio is relatively high. The signal subspace approach was exploited to form prior information for evoked potential estimation in a Bayesian setting [1]. In [2], a model based signal subspace was used for estimation. The subspace method has also been combined with a Wiener filter applied prior to SVD [3]. In [4] the data were first divided into signal and noise subspaces. Then the enhanced signals were further processed using lift wavelet transform. SVD was also used in [5] to form an observation model for dynamical estimation of EPs.

In this paper, an alternative approach is considered. Subspace based enhancement for a single sweep EP can be achieved by using time-delayed versions of the noisy signal. Furthermore, the ensemble SVD and the time delayed SVD can be combined. The resulting method outperforms ensemble SVD for single-trial EP estimation. The new method is demonstrated with real EP measurements obtained form an auditory experiment based on a habituation paradigm.

2. Methods

SVD has many theoretical and practical applications in signal processing and identification problems [6]. In relatively high signal-to-noise ratio conditions, it can divide measurements into signal and noise subspaces. Alternatively, it can also be understood in terms of principal component regression as a combined method for signal enhancement and optimal model dimension reduction [7].

The measured single-trial EP, sampled relative to the stimulus *t*, can be described through an additive noise model as

$$z_t(n) = s_t(n) + v_t(n),$$

for $t = 1, \dots, T$, $0 \le n \le N-1$. Where T is the total number of single trials and N is the epoch length. The signal s_t corresponds to the part of the activity that is related to the stimulation, and the rest of the activity v_t is assumed to be independent of the stimulus and the EP. Single-trial EPs can be further modeled as a linear combination of some basis vectors. Then the observation model in a vector notation becomes

$$z_t = H\theta_t + v_t,$$

where z_t is a column vector, and H is the observation matrix, which contains the basis vectors $\psi_1, \psi_2, \dots, \psi_k$ of length N in its columns. θ_t is a parameter vector of length k. The estimated EPs $\hat{s}_t$ can then be obtained by using the estimated parameters as

$$\hat{s}_t = H\hat{\theta}_t.$$

The available data matrix

$$Z = [z_1, z_2, \dots, z_t, \dots, z_T] \in \mathbb{R}^{N\times T},$$

which has as columns the epochs (vectors) sampled relative to the stimulation, can be decomposed as

$$Z = U\Sigma V^T,$$

where $U \in \mathbb{R}^{N\times N}$ satisfies $U^T U = I$, $V \in \mathbb{R}^{T\times T}$ satisfies $V^T V = I$, and $\Sigma \in \mathbb{R}^{N\times T}$ is a pseudo-diagonal matrix with non-negative diagonal elements σ_i such that $\sigma_1 \ge \sigma_2 \ge \dots \ge \sigma_{\min(N,T)} \ge 0$. If $N \le T$, then Σ has the form $\Sigma = [\Sigma_1\ 0]$, where $\Sigma_1 = \mathrm{diag}(\sigma_1, \dots, \sigma_N)$ and 0 is a zero matrix. If $N > T$, then Σ has the form $\Sigma = \begin{bmatrix}\Sigma_1 \\ 0\end{bmatrix}$, where $\Sigma_1 = \mathrm{diag}(\sigma_1, \dots, \sigma_T)$. Only r singular values are non-zero, where $r = rank(Z)$.

For the additive noise model and relatively small noise the following decomposition can be considered

$$Z = [U_s\ \ U_v]\begin{bmatrix}\Sigma_s & 0 \\ 0 & \Sigma_v\end{bmatrix}\begin{bmatrix}V_s \\ V_v\end{bmatrix}.$$

The matrix Σ_s contains the k largest singular values and U_s the respective left singular vectors associated mainly with the signals s_t. Thus the matrices $\{U_s, \Sigma_s, V_s\}$ represent the signal subspace, and $\{U_v, \Sigma_v, V_v\}$ represent primarily the noise subspace. From the SVD of the ensemble matrix we also have

$$ZZ^T = U\Sigma_1^2 U^T.$$

This means that the left singular vectors of Z are the eigenvectors of the ensemble data correlation matrix

$$\hat{R} = \frac{ZZ^T}{T}.$$

If we denote with H_s the matrix with columns the k dominant eigenvectors, then the ordinary least-squares estimator for the parameter vector becomes

$$\hat{\theta}_t = (H_s^T H_s)^{-1} H_s^T z_t = H_s^T z_t.$$

Alternatively, in a matrix formulation the estimated EPs are given by

$$\hat{S} = H_s H_s^T Z.$$

Quantitatively, the first basis vector is the best mean-square fit of a single waveform to the entire set of epochs. Thus, the first eigenvector is similar to the mean of the epochs, and the corresponding parameters $\hat{\theta}_t(1)$ $(t = 1, \dots, T)$ reveal the contribution of the eigenvector to each epoch. The rest of the dominant eigenvectors model primarily amplitude differences between individual EP peak components, and latency variations from trial-to-trial.

Subspace de-noising can also be applied when only one measured signal z_t is available [8]. This can be achieved by introducing $2p+1$ delay versions of the noisy signal into a Toeplitz matrix

$$Z_t = \begin{bmatrix} z_t(p) & z_t(p-1) & \dots & z_t(-p) \\ z_t(p+1) & z_t(p) & \dots & z_t(-p+1) \\ z_t(p+2) & z_t(p+1) & \dots & \vdots \\ \vdots & z_t(p+2) & \dots & z_t(p) \\ \vdots & \vdots & \dots & z_t(p+1) \\ z_t(-p+N-1) & \vdots & \vdots & \vdots \\ \vdots & z_t(-p+N-1) & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ z_t(p+N-1) & z_t(p+N-2) & \dots & z_t(-p+N-1) \end{bmatrix},$$

where the columns contain the time-shifted vectors, and $Z_t \in \mathbb{R}^{N\times(2p+1)}$. Note that a longer measurement epoch is now needed. As before, from the SVD of the matrices Z_t, i.e. $Z_t = U_t\Sigma_t V_t^T$ for every t, by selecting k dominant left singular vectors to form the columns of the matrix H_{s_t}, single-trial estimates can be obtained as

$$\hat{s}_t = H_{s_t} H_{s_t}^T z_t.$$

Again relatively small noise is assumed.

A combination of ensemble and time-shifted SVD is here proposed for single-trial EP estimation. A larger measurement matrix can be formed by considering ensemble and within individual epochs statistical information as follows

$$\boldsymbol{Z} = [Z_1\ \ Z_2\ \ \dots\ Z_t\ \ \dots\ Z_T] \in \mathbb{R}^{N\times T(2p+1)},$$

where Z_t contains time-shifted versions of the sampled potential z_t in each column. Similarly, from the SVD of $\boldsymbol{Z}$, i.e. $\boldsymbol{Z} = \boldsymbol{U\Sigma V}^T$, by selecting k dominant left singular vectors to form the columns of the matrix $\boldsymbol{H}_s$, single-trial estimates can be obtained as

$$\hat{s}_t = \boldsymbol{H}_s \boldsymbol{H}_s^T z_t,$$

or in a matrix notation as

$$\hat{S} = \boldsymbol{H}_s \boldsymbol{H}_s^T Z.$$

3. Results

In this section, the proposed single-trial estimation method is evaluated with real EP measurements obtained with an auditory habituation paradigm. Trains of four stimuli were presented to the subject via headphones. In each stimuli train, the inter stimulus interval was 1.1 s. In every stimulation, the delivered tones were the same (frequency, duration, intensity).

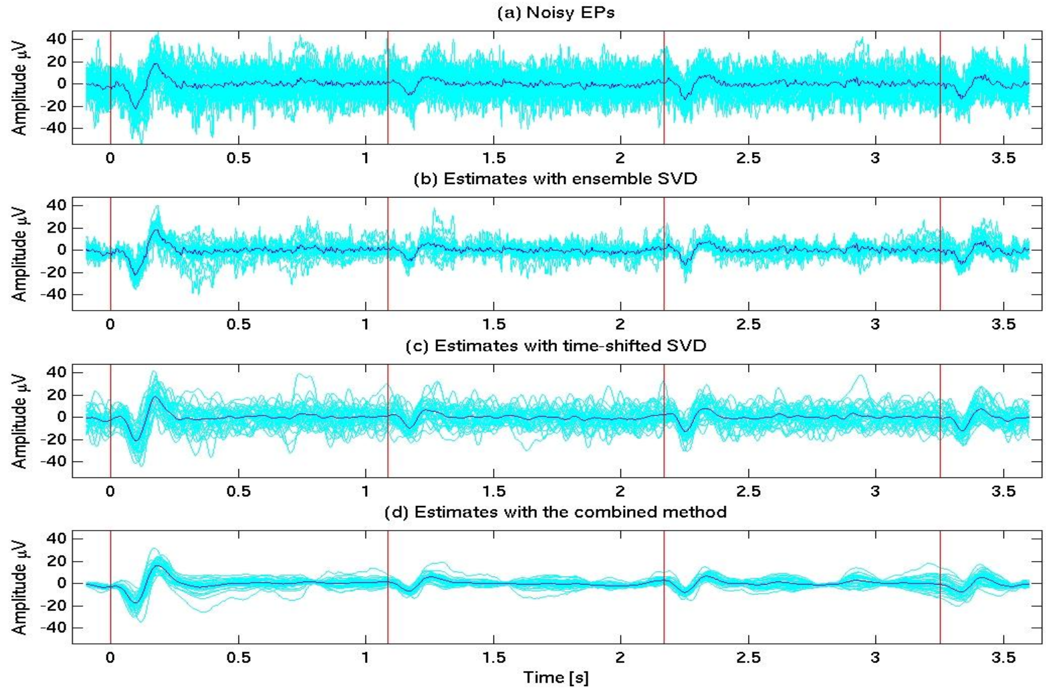

Figure 1: (a) Noisy EEG epochs, (b) estimates obtained with ensemble SVD (k = 4), (c) estimates obtained with time-shifted SVD (k = 4) for each epoch individually, and (d) estimates obtained with the combined approach (k = 4). Vertical lines denote stimuli presentation time. Dark lines represent the average of the epochs in each case.

The stimulation was repeated every 12 s. A total of 32 stimuli trains were presented to the subject. From the recordings we used the channel Cz after high pass filtering at 1 Hz. The sampling frequency was 500 Hz.

For estimation, we selected epochs by concatenating a whole stimuli train in a single vector. Thus, 32 noisy signals were used (t = 1,...,32), where each covers in time a whole stimuli train. The measurements and their average vector are presented in Fig. 1 (a). In this plot it is visible the N100/P200 auditory complex. Furthermore, a decrease in amplitude is observed after the first stimulus in row.

Estimates obtained with the subspace method applied on the ensemble measurement matrix are presented in Fig. 1 (b). For estimation, 4 dominant left singular vectors were used, i.e. k = 4. In the same figure (Fig. 1 (c)), estimates based on the time-shifted SVD are presented. Time-shifted SVD was applied to every epoch individually, and again 4 dominant singular vectors were used. Finally, in Fig. 1 (d) estimates obtained from the combined approach, with k = 4 are presented. Clearly, better estimates are obtained using the combined approach, with greater noise reduction.

The estimated signal subspace in each method is presented in Fig. 2. Note that the time shifted SVD is applied in every epoch separately. Therefore, in this plot two randomly selected epochs are presented ((b) and (c)). In Fig.2 (d) the eigenvectors from the combined approach are presented, which clearly contain less noise. This has as a consequence the provision of better EP estimates.

Single-trial EP estimates can be used to extract higher statistical information than the mean from a single subject. From the estimates, the amplitude and latency of the N100 peaks were obtained. In each single-trial waveform, the 4 consecutive peaks were estimated as the minimum values between 0-200 ms from each stimulus presentation time. The estimated amplitudes and latencies are presented in Fig. 3 as a box and whisker plot. The boxes have lines at the lower quartile, median, and upper quartile. Whiskers extend from the box to the most extreme value within 1.5 i, where i is the inter-quartile range. Boxes whose notches do not overlap indicate that the medians of the two groups differ at the 5% significant level. Here, the decrease in amplitude is more visible. Furthermore, it can be observed that the presentation of the first stimuli of the trains result to a slower response. Both are manifestations of the habituation effect.

4. Conclusion

In this paper, a method for single-trial EP estimation was presented. The method is based on the estimation of the EP signal subspace from a combination of time-shifted and ensemble measurements. Computationally this is achieved through SVD. The method outperforms the ensemble SVD approach for single-trial EP estimation.

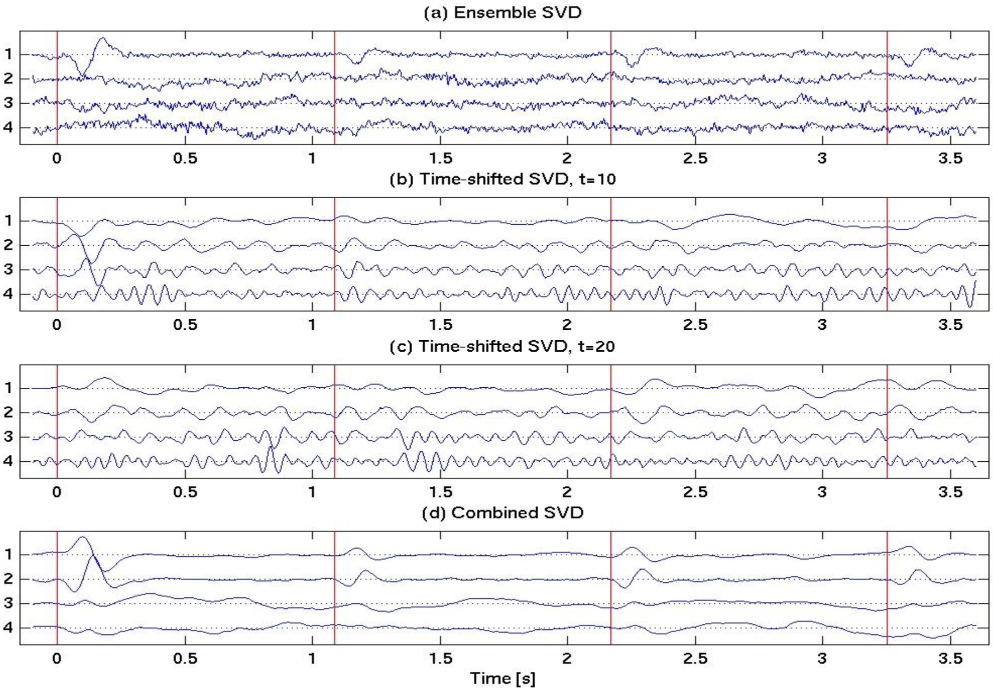

Figure 3: Left singular vectors (4 dominant) (a) obtained from the ensemble data, (b) with time shifted SVD for t = 10, (c) with time-shifted SVD for t = 20, and (d) with the combined approach. Vertical lines denote stimuli presentation time.

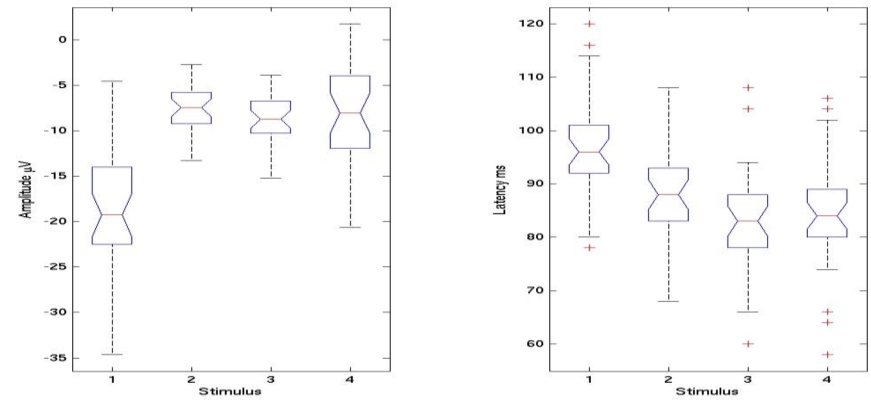

Figure 2: Box whisker plots for the estimated amplitudes and latencies of the four stimuli at the train.

5. References

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