RECURSIVE BAYESIAN ESTIMATION OF SINGLE TRIAL EVOKED POTENTIALS

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Abstract— **A method for the sequential estimation of single trial evoked potentials (EP) is presented. The method is based on recursive Bayesian Mean Squares estimation. The evoked potentials are estimated sequentially using the old estimates as the prior information. The estimated EP's are constrained to a principal subspace of the ensemble of measurements. The method is shown to be capable of tracking slow trends in parameters of the EP. The performance of the method is evaluated with realistic simulated evoked potential measurements.**

I. Introduction

Over three decades it has been evident that amounts of useful information is lost when the evoked potentials (EP) are averaged [1]. This is caused by the fact that the components of the potentials are not always in coherence between the repetitions of the stimulus. Variations in the latency and the amplitude of the components cause that the averaged evoked potential can differ very much from each single potential and does not correspond to any possible physical situation. Of special interest is the case, when some parameter of the EP changes slowly from stimulus to stimulus. This kind of the situation can be e.g. a trend such as the change of the latency of some specific component potential of the EP.

One approach to extract information beyond the average has been the filtering of the single trials of measurement. Time-varying Wiener filtering has been the approach that exhibits the most realistic assumptions about the signal [2], [3]. The time varying Wiener-filter is known to be the linear minimum mean square estimator for random parameters [4].

The result of the filtering is thus an estimate for the underlying evoked potential signal, which is measured in presence of noise. The performance of the estimator or the filter is naturally dependent on the prior information about the statistical properties of the signal and the noise. If we assume for example that the evoked potential is a vector valued random process with slow variations between the repetitions, the evoked potentials can be feasibly described with the aid of a state space model. The recursive mean square solution for this can be calculated with the Kalman-filter [4].

Several models have been used for single evoked potential measurements in different studies. The most common ones have been different types of Gaussian shaped components and exponentially damped sinusoidal functions [3]. The task has then been to fit these basis functions to ensemble of measurements in the least squares sense. If we do not restrict these basis functions to be any specific elementary functions and model the EP as linear combination of some basis functions, it is well known that the eigenvectors of the covariance of the ensemble of the EP's is the choice, that minimizes the mean square error. This approach is also called the Principal Component Analysis (PCA) approach [5].

In this study our goal is to form an estimator that meets the following requirements:

1. Capability to track slow trends in the parameters of the EP.
2. Nonparametric nature of the estimator.
3. Full use of the second order statistical information available in ensemble of measurements.

The first requirement is met by using the Kalman filter for recursive estimation of the EP's. The second and the third requirements are met when we model the EP's with the PCA approach. Then the EP's are modeled as linear combination of eigen vectors, which are obtained without any functional (parametric) assumptions about the EP's.

Finally we use the method called *smoothness priors* for the smoothing of the individual EP estimates [6].

II. Methods

Assume that the vector of the random parameters θ is to be estimated using another random vector z. If we restrict the estimator to be a linear mapping from z to θ it can be shown that the estimator that is best in mean square sense is

$$\hat{\theta}_{\mathrm{MS}} = C_{\theta z} C_z^{-1} z \tag{1}$$

where C_z is the covariance of the measurements and $C_{\theta z}$ is the cross-covariance of the measurements and the parameters to be estimated. The estimator is optimal among all possible estimators, not necessarily linear, if θ and z are jointly Gaussian [4]. This form is independent of the model of how the observations z and parameters θ are related, but in practice we cannot calculate $C_{\theta z}$ without assuming a model for the observations.

In many cases we can use linear model for the observations

$$z = H\theta + v \tag{2}$$

where v is a random vector and H is a deterministic observation matrix. Now the linear mean square estimator can be written in the form

$$\hat{\theta}_{\mathrm{MS}} = (H^T C_v^{-1} H + C_\theta^{-1})^{-1}(H^T C_v^{-1} z + C_\theta^{-1}\eta_\theta) \tag{3}$$

where C_θ and η_θ are the covariance and the mean of θ respectively [4]. They present the prior information about the parameters θ. C_v is the covariance of the measurement noise. If they are not available, we can make $C_\theta^{-1} = 0$, and the estimator reduces to the ordinary minimum variance estimate. Next we assume that the parameters obey the time variation with so called random-walk model

$$\theta_{t+1} = \theta_t + w_t \tag{4}$$

where w_t is a random vector. Equations (4) and (2) form together the so-called state space equations. It is well known that the recursive mean square estimate for θ_t is given by Kalman-filter. It is in fact the recursive linear Bayesian mean square estimate for θ_t using previous estimate as prior information. With this model the Kalman-filter takes the form

$$\begin{aligned} K_t &= P_t H^T (H P_t H^T + C_v)^{-1} && (5)\\ P_{t+1} &= (I - K_t H) P_t + C_w && (6)\\ \hat{\theta}_{t+1} &= \hat{\theta}_t + K_t(z_t - H\hat{\theta}_t) && (7) \end{aligned}$$

where K_t is the so-called Kalman -gain vector and $\hat{\theta}_t$ is a time-varying estimate for the parameters θ_t.

The most often used observation model in context of evoked potential analysis is

$$z = s + v \ . \tag{8}$$

This is of the form of (2) with the identity matrix as the obervation matrix $H = I$ and the parameter vector $\theta = s$ equals the evoked potential. The random vector v is then the ongoing background EEG activity. The equation (1) is then exactly the time-varying Wiener-filter used e.g. in [2], [3]. The evoked potential s is often further parametrized with some functional form for estimation of the covariances.

We make here another assumption. We express the EP's as linear combinations

$$s = K_{\mathcal{S}}\theta \tag{9}$$

where the columns of $K_{\mathcal{S}}$ are the eigenvectors of R_s, the correlation matrix of the EP's

$$R_s = E\left\{ss^T\right\} \ . \tag{10}$$

R_s is then an outer product of ensemble of rank one matrices. The realizations s span a vector space $\mathcal{S}$. In EP case $\mathcal{S}$ is spanned by those vectors which are possible outcomes of the experiment. Even if the outcome of the test is thought to be random, $\mathcal{S}$ can be approximated well with some low dimensional subspace of it. Usually only few eigenvectors corresponding the few largest eigenvalues are needed. In the observation model (2) we put then $H = K_{\mathcal{S}}$. This approach also reduces the dimensionality of the problem from few hundreds (the length of the observation) to few parameters.

At last step of the analysis the estimated potentials are calculated from parameters. This calculation can be done with simultaneous *smoothness priors* –smoothing with the equation

$$\hat{z}_t = (I + \alpha L^T L)^{-1} H\hat{\theta}_t \tag{11}$$

where L is the second difference matrix and α is a smoothing factor.

III. Results

We used the method for estimating the trials of the simulated evoked potentials. The simulated potentials had two negative and one positive component potential. The latencies and amplitudes were uniformly distributed random numbers. As background EEG we used realizations of an AR(6) process. The parameters of the process correspond to realistic EEG process. Ten randomly selected simulations are shown in Fig. 1. The linear trend was removed from every simulation. The simulations were then sorted according to the latency of the positive component to simulate a slow trend in latency.

Next we calculated the eigenvectors of the covariance matrix of the measurements. The matrix $K_{\mathcal{S}}$ can be approximated with these well. We used the dimension of 5 for the subspace $\mathcal{S}$.

The Kalman-filter algorithm (5)-(7) and the *smoothness-priors* –estimate (11) were then applied to simulations. The estimated EP's are shown in Fig. 3. These can be compared to the noiseless simulations shown in Fig. 2. We were interested in the tracking capability of the algorithm and thus calculated the maximum of the positive component of the estimated EP's. The time point of the maximum is plotted as a function of latency in Fig. 4. A typical estimate is shown in Fig. 5 together with the true EP and the simulation. The vertical lines are the true and the estimated latency.

IV. Conclusions

We have presented a method for recursive estimation of the evoked potentials when the parameters of the potentials have trends. In the presented algorithm the Kalman-filtering and the principal component approaches are combined with the *smoothness priors* –approach. The

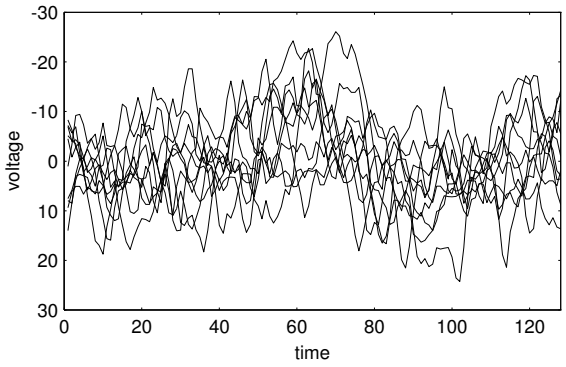

Fig. 1. Ten simulations of the EP's. Time is in points and voltage is in microvolts.

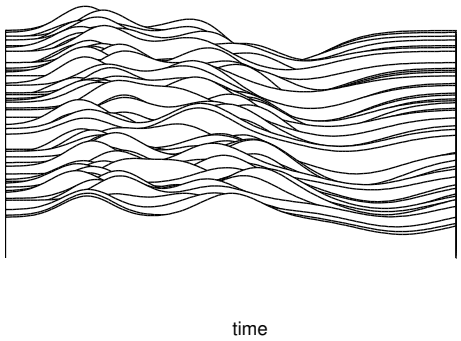

Fig. 2. Ensemble of EP simulations without noise. The positive potential axis is downwards.

simulations show that the algorithm is capable of tracking slow trends in evoked potential parameters. Note that in this example the negative components varied randomly and did not fulfill the assumption of trend-like variation. In tests the algorithm has been provided to be robust and reliable also for real EP measurements. The method can be further developed by using so called hypermodels for the state variations [7].

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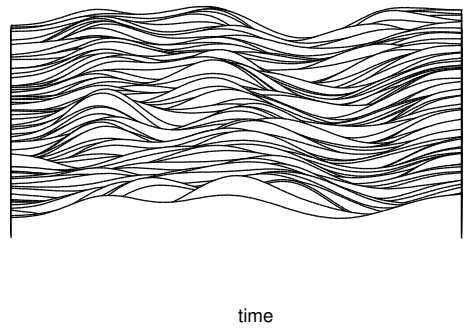

Fig. 3. Estimated EP's. The positive potential axis is downwards.

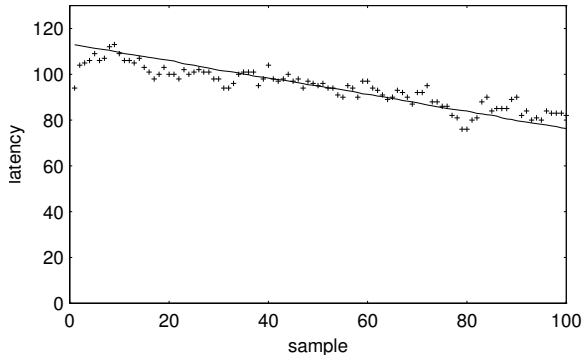

Fig. 4. The true and the estimated latency in points as a function of the simulation number.

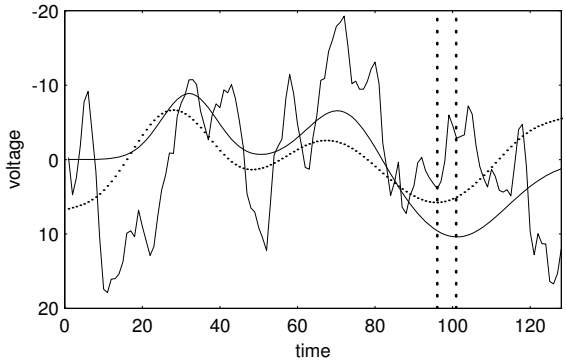

Fig. 5. Typical simulation (solid, rough) and estimate (dotted) of the EP with the noiseless simulation (solid, smooth). The vertical lines are the estimated an the true latency.