TRACKING OF NONSTATIONARY EEG WITH THE POLYNOMIAL ROOT PERTURBATION

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Abstract— **A method for estimation of the change point in event related desynchronization test is presented. The method is based on tracking of a single system pole of the time-varying ARMA model. The pole is approximated using perturbation theory.**

I. Introduction

The end to automatic analysis of EEG is often segmentation of the EEG into stationary epochs. The parameters of autoregressive (AR) and autoregressive moving average (ARMA) models have been found to exhibit reasonably good discrimination efficiency in many cases [1]. Several algorithms exist for calculation of the model parameters. Since EEG is a nonstationary signal, a natural choice is to use an adaptive algorithm, such as the recursive least squares (RLS) algorithm [2].

Sometimes the model roots are more suitable for making further inference than the model parameters directly [3]. The use of model roots has been proposed earlier for the classification of stationary epochs of EEG [4]. The calculation of all the roots from model parameters each time can be computationally too expensive if some standard method is used. The roots can, however, been approximated with the so-called eigenvalue perturbation theory.

We extend these results to the event related synchronization/desynchronization (ERS/ERD) analysis of alpha waves of EEG [5]. When a patient has his/her eyes closed, the occipital EEG shows high intensity in the 8–12 Hz region (synchronization) while with the opening of the eyes this intensity decreases or even vanishes (desynchronization). The end of the analysis is to find the transition point in EEG signal between the states.

II. Methods

Time-varying ARMA(p,q) models for the process x_t can be written as

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + \sum_{\ell=1}^{q} b_\ell(t) e_{t-\ell} + e_t \ , \tag{1}$$

where e_t is a prediction error process (white noise) and the parameters $a_k(t)$ and $b_\ell(t)$ are estimated with an adaptive predictor. There are several algorithms that can be used as predictors. We use here the recursive least squares (RLS) algorithm [2], which takes the form

$$X_t = [x_{t-1}, \ldots, x_{t-p}, e_{t-1}, \ldots, e_{t-q}]^{\mathrm{T}} \tag{2}$$

$$e_t = x_t - \theta_{t-1}^{\mathrm{T}} X_t \tag{3}$$

$$K_t = P_{t-1} \frac{X_t}{\mu + X_t^{\mathrm{T}} P_{t-1} X_t} \tag{4}$$

$$\theta_t = \theta_{t-1} + e_t K_t \tag{5}$$

$$P_t = \mu^{-1}(I - K_t X_t^{\mathrm{T}}) P_{t-1} \tag{6}$$

where

$$\theta_t = [\hat{a}_1(t), \ldots, \hat{a}_p(t), \hat{b}_1(t), \ldots, \hat{b}_q(t)]^{\mathrm{T}} \tag{7}$$

and the transpose is denoted by $(\cdot)^{\mathrm{T}}$. The trade-off between tracking speed and estimate variance is controlled via the forgetting factor $\mu < 1$.

We aim to approximate a root $\lambda_k(\epsilon)$ of the polynomial $\hat{a}(t_0) + \epsilon \delta \hat{a}(t)$ where $\delta \hat{a}(t) = \hat{a}(t) - \hat{a}(t_0)$ with the second order Taylor series expansion

$$\lambda_k(\epsilon) \approx \lambda_k + \epsilon \lambda_k' + \frac{1}{2} \epsilon^2 \lambda_k'' \ , \tag{8}$$

where λ_k is a root of $\hat{a}(t_0)$. As is well known, the roots of a polynomial equal the eigenvalues of the associated companion matrix

$$A = \begin{pmatrix} \hat{a}_1(t_0) & \hat{a}_2(t_0) & \ldots & \hat{a}_p(t_0) \\ 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix} \tag{9}$$

where all the blank entries are zeros. If $\hat{a}(t_0)$ is perturbed by $\delta \hat{a}(t)$, the new roots are the eigenvalues of $A + \epsilon B$, where $\epsilon = 1$, B is a $p \times p$ matrix with first row $(\delta \hat{a}_1(t), \ldots, \delta \hat{a}_p(t))^{\mathrm{T}}$ and all the other entries zeros. It can be shown [6], that the second order Taylor expansion for λ_k is now (denoting $\lambda_k(1)$ by $\lambda_k(\delta \hat{a}(t))$ to stress the dependence of the perturbation on $\delta \hat{a}(t)$ and setting $\epsilon = 1$)

$$\lambda_k(\delta \hat{a}(t)) = \lambda_k + \frac{w_k^{\mathrm{T}} B v_k}{w_k^{\mathrm{T}} v_k} + (w_k^{\mathrm{T}} v_k)^{-1} w_k^{\mathrm{T}} \left[B - \frac{w_k^{\mathrm{T}} B v_k}{w_k^{\mathrm{T}} v_k} I \right] \sum_{\substack{\ell=1 \\ \ell \neq k}}^{p} \frac{w_\ell^{\mathrm{T}} B v_k}{(\lambda_k - \lambda_\ell) w_\ell^{\mathrm{T}} v_\ell} v_\ell \tag{10}$$

where w_k and v_k are the left and right eigenvectors of A respectively. The special structure of B makes it possible to write (10) in a form in which the complexity of the approximation is only $2p+1$ complex multiplications and additions for a single root. The approximation capability of (10) is visualized in Fig. 1.

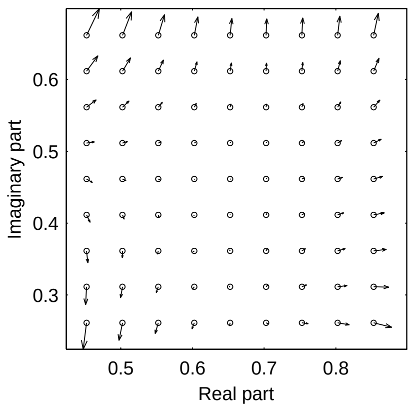

Fig. 1. The error in root approximation in a selected area of the complex plane. The expansion center $\hat{a}(t_0)$ is in the center of the grid. The grid envelope is also shown in Fig.3.

III. Results

We used the method for estimation of the change points in the event related desyncronization test. Three typical epochs are shown in Fig. 2.

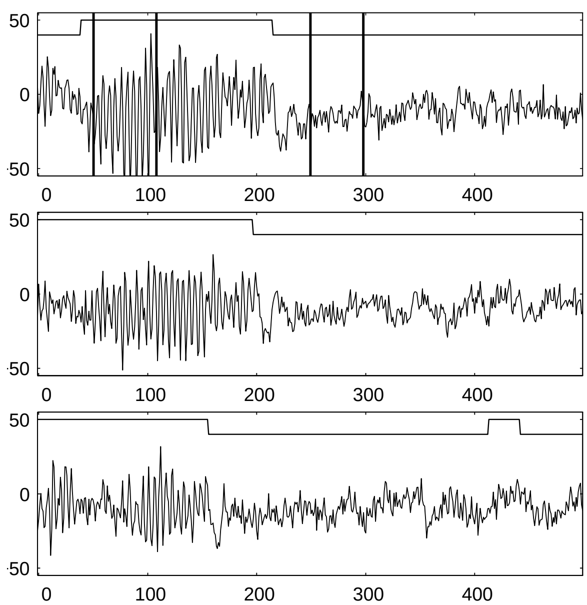

Fig. 2. Three typical epochs of ERD test. The vertical lines show the learning sets. The stair function denotes the result of the detection.

The procedure of the change point detection was as follows:

1. Select two segments from the data, one for each state (vertical dotted lines in Fig. 2). Use these as the learning sets for the classes.

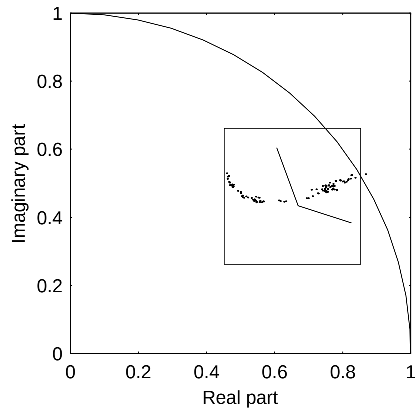

Fig. 3. The detection border is selected using the roots of the learning sets. The square denotes the approximation area of Fig. 1.

2. Run these segments through RLS to obtain parameter estimates for the classes.
3. Approximate the roots of interest with (10) and determine the detection boundary (Fig. 3).
4. Run the whole data through RLS and a root approximation procedure, classify and run further through an optional post-processor (e.g. a median filter).

The result of detection is visualised in Fig. 2 with stair functions.

IV. Conclusions

The two states of the event related desyncronization test can be efficiently discriminated by a single pole of an ARMA(6, 2) model. The tracking of the model parameters can be done with the forgetting factor RLS algorithm and the root of interest can be approximated with the perturbation equations. The computational burden is small enough to allow for a real time implementation with a general purpose personal computer.

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