# ESTIMATION OF THE DYNAMICS OF MEDIUM RATE EEG TRANSITIONS

J.K. Hiltunen[†*], P.A. Karjalainen[†], J.P. Kaipio[†]
[†]Dept. of Applied Physics, University of Kuopio, P.O.Box 1627, FIN-70211, Kuopio, FINLAND
and [*]Dept. of Clinical Neurophysiology, Kuopio University Hospital, P.O.Box 1777,
FIN-70211, Kuopio, FINLAND, E-mail: kaipio@venda.uku.fi

*Abstract*— **A method for the estimation of medium rate transitions of nonstationary EEG is presented. The method is applicable to such EEG dynamics that are between a) fast transitions for which segmentation procedures are used and b) slow transitions for which adaptive filters work properly. The estimation of the transition dynamics is based on a novel time-varying autoregressive model. The performance of the method is evaluated with realistic simulations of known transition dynamics.**

*Keywords*— **EEG, ERD, time-varying autoregressive model, estimation.**

## I. Introduction

The traditional approach in the analysis of background EEG has been the segmentation of EEG into (almost) stationary epochs. This approach works often reasonably well in both the very fast and very slow transition cases. In the former case a segmenter and in the latter case an adaptive algorithm can be used. However, there are only few algorithms for the estimation of such transitions whose rate falls between these two limiting cases.

Recently there has been increasing interest in the transition dynamics of EEG, that is, how the change occurs – not only the initial and final steady states. The so-called ERD/ERS test (for definition, see below) is an example of an application in which the transition has been investigated for clinical relevance [1], [2] and methods for this task have been developed, e.g. in [3]. The ERD/ERS can be briefly described as follows.

Stimulus–induced blocking or attenuation of rhythms within the alpha frequency band (8–12 Hz) is called event–related desynchronization (ERD). The opposite phenomenon, event–related synchronization (ERS), is the increase of rhythms within the alpha band [4]. Both ERD and ERS have been widely used in the assessment of lesions and neurophysiological pathologies such as strokes and tumors [5]. However, also in these cases mainly the steady states before and after the transitions have been examined for the clinical evaluation. It is also believed that the there might be different delays in the occurrence of the desynchronization after the stimulus, and that these delays might convey some information on the neurophysiological state.

The statistical properties of most previously used estimation methods do not enable the estimation of a single sample. However, if for example trends in the possible delays are to be detected we have to be able to estimate the individual responses with adequate accuracy.

An algorithm for the estimation of such transition dynamics in EEG was proposed in [6]. This algorithm is based on the basis constrained least squares time-varying autoregressive model (TVARLS). The model works relatively well in the estimation of the transition dynamics but it has the drawback that it has to initialized for each patient separately. This initialization needs supervision and is computationally burdensome.

In this paper we propose an algorithm for the estimation of medium rate EEG transition dynamics that is based on a modification of the TVARLS algorithm. This modification reparametrizes the TVARLS problem so that a single parameter is more or less linearly dependent on the transition instant of the EEG.

This paper is organized as follows. In Section II-A the basis constrained time–varying autoregressive model is formulated and the basis selection problem is discussed in Section II-B. The modification of the TVARLS problem is described in Section II-C and is applied to simulated EEG in Section III. Finally, in Section IV we discuss the proposed method.

## II. Methods

### A. The general TVARLS model

The time-varying AR($p$) model (TVAR) is generally of the form

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t \tag{1}$$

where $p$ is the order of the model, $e_t$ are the residuals and $a_k(t)$ are the time–varying prediction coefficients. In the basis constrained TVAR problem the coefficients are restricted as

$$a_k(t) = \sum_{\ell=0}^{M} c_{k\ell} \phi_\ell(t) \ , \tag{2}$$

where $\phi_\ell(t),\ \ell = 0, \ldots, M$ are the basis functions.

The minimization of the 2-norm of the residuals leads to a quadratic problem in the $(M+1)p$ parameters $c_{k\ell}$ and the least squares solution can be accessed by writing $C = (c_{10}, \ldots, c_{1M}, \ldots, c_{p0}, \ldots, c_{pM})^{\mathrm{T}}$, $X =$

$(x_{p+1},\ldots,x_T)^{\mathrm{T}}$, $E = (e_{p+1},\ldots,e_T)^{\mathrm{T}}$ and regressor matrix $H = (H_{10},\ldots,H_{1M},\ldots,H_{p0},\ldots,H_{pM})$ where $H_{k\ell} = (\phi_\ell(p+1)x_k,\ldots,\phi_\ell(T)x_{T-p+k})^{\mathrm{T}}$ and $(\cdot)^{\mathrm{T}}$ denotes transpose. The least squares problem can now be stated as

$$\min_C \|X - HC\|_2 \ , \tag{3}$$

the solution of which is

$$C = (H^{\mathrm{T}}H)^{-1}H^{\mathrm{T}}X \ . \tag{4}$$

The time-varying coefficients are then assembled via (2). One of the problems that are associated with the TVARLS method is that all coefficients are structurally independent and thus, for example, the transitions in the coefficient evolutions can occur at very different times. Thus inference that is based on the coefficients usually becomes ambiguous.

The method that was described in [6] forces the coefficient transition times to occur simultaneously but has the drawbacks that were mentioned in Section I.

### *B. Selection of basis functions for transition dynamics*

Several sets of functions have been used with the TVARLS model and the selection of basis functions has been discussed widely in the literature [3], [7]. Basis functions that are commonly used to model smooth changes include polynomial, Fourier and discrete spheroidal bases.

However, with these generic bases the parameter estimation problem has too many degrees of freedom in the case of the ERD/ERS transition. In this case we can usually assume that the dynamics can be described as a transition from a state to another. With this presumption we can tailor the basis functions so that they allow changes only in the assumed transition region. Such a set of basis functions can be constructed as in [3] in which a set of sigmoidal transition models for the coefficients was constructed and an optimal low-dimensional approximating set was determined.

We use this approach also in this paper and use two basis functions and add the constant function. The chosen set of basis functions is shown in Fig. 1. Let the coefficient $c_{k1}$ be positive. It can then clearly be seen that if $c_{k2}$ is also positive, the transition occurs earlier than in the case in which $c_{k2}$ is negative. Thus this selection of basis functions is structurally able to model the instant of a transition for each of the coefficient evolutions separately, see also Fig. 2.

### *C. The β–parametrization of the TVARLS problem*

Our aim is to reparametrize the TVARLS problem so that the transition instant can be modelled with a single parameter. We have now $M = 2$ and the case $c_{k2} = 0$ corresponds to a situation that the transition occurs in the middle of the interval.

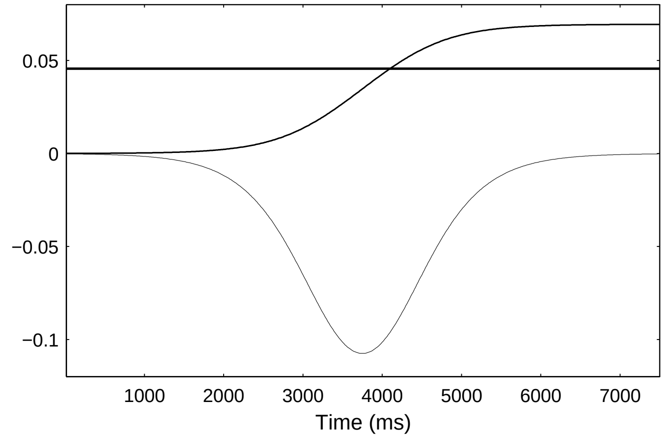


Fig. 1. The constant vector and the two first eigenvectors of the covariance of the set of sigmoids, $\varphi_\ell(t)$, $\ell = 0, 1, 2$.

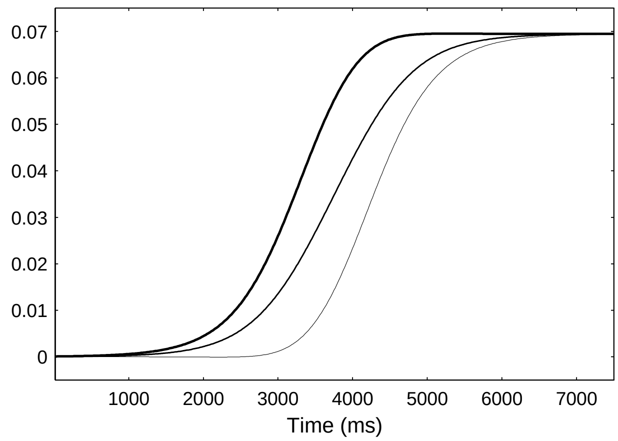


Fig. 2. The dependence of a coefficient evolution $a_k(t) = \sum_{\ell=0}^{2} c_{k\ell}\varphi_\ell(t)$ of the parameter $\beta$. Here $c_{k0} = 0$, $c_{k1} = 1$ and $c_{k2} = \beta c_{k1}$. Bold line: $\beta = 0.2$, medium line: $\beta = 0$ and weak line: $\beta = -0.2$.

We reparametrize the problem so that we apply the constraints

$$c_{k2} = \beta c_{k1}, \quad \text{for all } k = 1,\ldots,p \ . \tag{5}$$

We have thus $C = (c_{10}, c_{11}, \beta c_{11},\ldots,c_{p0}, c_{p1}, \beta c_{p1})^{\mathrm{T}}$ and the transition instants of each coefficient evolution coincide. The numerical solution to this problem is given in the Appendix.

Thus the transition instants are constrained to occur simultaneously for all coefficients and this instant depends only on the single parameter $\beta$. The dependence of the transition can be seen Fig. 2. The smaller the $\beta$, the earlier the transition.

## III. Experimental results

We constructed a set of 14 different time-varying AR(6) processes as in [8]. These processes simulate ERD transitions that occur smoothly. The dynamics of the simulations

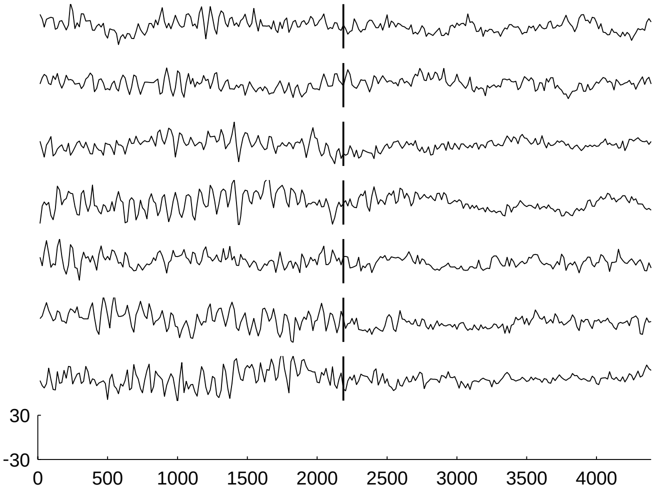


Fig. 3. Seven examples of simulated time-varying EEG (ERD). The evolution of coefficients in this model is the same for all simulations. The vertical lines denote the center of the transition region.

is realized with Gaussian-shaped basis and is thus different from the basis that is used in the estimation. For each of the processes 10 realizations at 11 different transition instants were simulated resulting in 1540 cases of ERD. The $\beta$ parameter was estimated for each of these ERD simulations, see Fig. 3 for 7 examples of the simulations.

To obtain the dependence of $\beta$ on the transition, we construct the mixed norm problem

$$\min_{\Psi} \left\{ \sum_{jk\ell} |T_{\rm d}(j) - \psi_1 - \psi_2 \beta_{jk\ell}|^1 + \alpha \sum_{j} |T_{\rm d}(j) - \psi_1 - \psi_2 \bar{\beta}_j|^2 \right\} , \tag{6}$$

where $\alpha = 1000$, $\Psi = (\psi_1, \psi_2)^{\rm T}$, $\beta_{jk\ell}$ is the estimate for the $j$'th delay of the $k$'th realization of the $\ell$'th process and $\bar{\beta}_j$ is the mean over all realizations and processes with delay $T_{\rm d}(j)$.

The time delay estimate is of the form $\widehat{T}_{\rm d} = \psi_1 + \psi_2 \beta$. Thus the minimization of (6) tries to achieve a balance between the delay errors for each of the 1540 cases and the linearity of the delay estimate $\widehat{T}_{\rm d}$. The mean delay estimate is shown in Fig. 4 and an example of the delay tracking characteristics in Fig. 5.

The duration of the transition period of the true coefficient evolutions is approximately 800 ms. Examples of the tracking of the transitions are shown in Fig. 6. The mean error for the transitions is less than one third of the duration of the transition. Thus the proposed method is able to distinguish the (predefined) transition instant even within the transition period. The detection of this instant within a set of realizations from a particular process is problematic for segmentation algorithms.

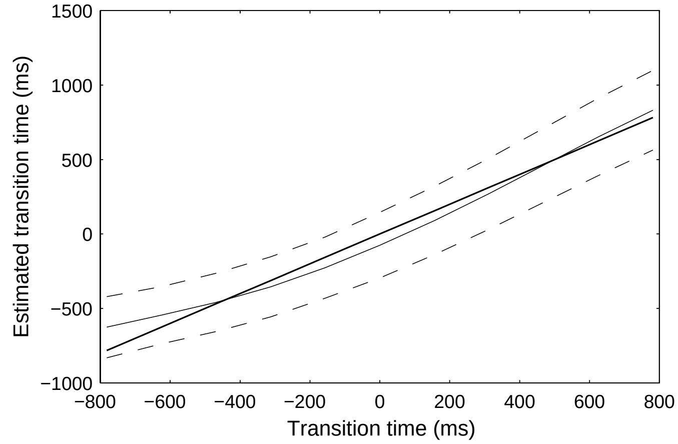


Fig. 4. The true delay $T_{\rm d}$ (bold), the mean of the estimated delays $\widehat{T}_{\rm d}$ (weak) and the mean error intervals.

## IV. Discussion

We have proposed a method for the modelling of smooth medium rate transitions of EEG. The method is regularized so that the transitions of all coefficient evolutions occur simultaneously. Thus the model is a restriction of the general TVARLS model and it can be used i such cases in which the rate of transition is approximately known but the instant of transition is not known. The general TVARLS model can describe more complicated variations is EEG transitions such as the rate of transition.

When the applicability of the proposed method is compared to the catenated-coefficient parametrization of the TVARLS model [3], [6], the main difference are that a) the proposed method is structurally less sensitive to the initial and final states and b) the proposed method can not estimate rates of transition.

The performance of the method was investigated with simulations that conform to the assumptions on the rate of the dynamics. The method was shown to be able to track the instant of a smooth medium rate transition. Such transitions conflict with the assumptions that are inherent when conventional segmenters and adaptive algorithms are used.

## Appendix

### I. Iterative solution to the $\beta$–parametrization problem

The notations are as in Section II-A. The term $HC$ can be expanded

$$\begin{aligned} HC &= H_{10}c_{10} + H_{11}c_{11} + H_{12}\beta c_{11} + \cdots + \\ &\qquad H_{p0}c_{p0} + H_{p1}c_{p1} + H_{p2}\beta c_{p1} \qquad (7) \\ &= \sum_{k=1}^{p} (H_{k0}c_{k0} + H_{k1}c_{k1} + \beta H_{k2}c_{k1}) \qquad (8) \end{aligned}$$

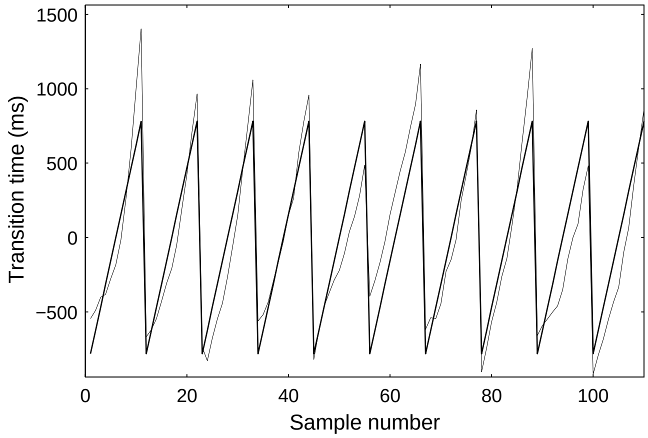


Fig. 5. The true delays $T_\mathrm{d}$ (bold) and the estimated delays $\widehat{T}_\mathrm{d}$ (weak) for one evolution, 11 delays and 10 realizations shown consecutively.

$$\doteq \quad F(\alpha) \ , \tag{9}$$

where $\alpha = (c_{10}, c_{11}, \ldots, c_{p0}, c_{p1}, \beta)^\mathrm{T}$.

This mapping $\alpha \mapsto F(\alpha)$ is nonlinear and thus the obtained least squares problem

$$\min_\alpha \|F(\alpha) - X\|_2 \tag{10}$$

is nonquadratic and has to be solved iteratively. We solve this problem with the Levenberg-Marquardt method (stabilized Gauss-Newton algorithm).

The Jacobian $J_F(\alpha)$ of $F(\alpha)$ is obtained by direct calculation and is given columnwise by

$$J_F(\alpha) = \Bigg( H_{10}, (H_{11} + \beta H_{12}), \ldots, H_{p0}, (H_{p1} + \beta H_{p2}), \sum_{k=1}^{p} H_{k2} c_{k1} \Bigg) \in \mathbb{R}^{T-p \times 2p+1} \ . \tag{11}$$

The Levenberg-Marquardt algorithm takes the form [9]

$$\alpha^{(\ell+1)} = \alpha^{(\ell)} + \Big( J_F^\mathrm{T}(\alpha^{(\ell)}) J_F(\alpha^{(\ell)}) + \gamma I \Big)^{-1} J_F^\mathrm{T}(\alpha^{(\ell)}) \Big( X - F(\alpha^{(\ell)}) \Big) \ , \tag{12}$$

where $\gamma > 0$ is a (small) stabilization parameter and $I$ is the identity matrix. Normally 10 iterations are adequate to achieve sufficient accuracy.

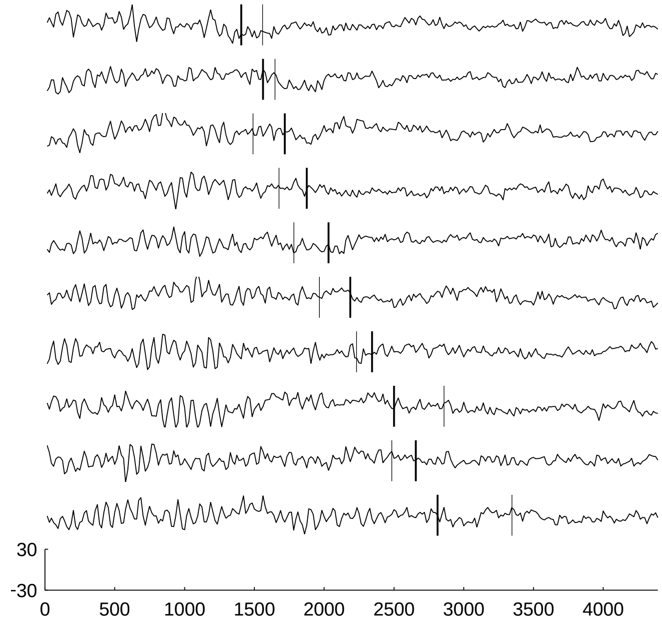

Fig. 6. Delay estimates for a process with different delays, true delays $T_\mathrm{d}$ (bold) and the estimates $\widehat{T}_\mathrm{d}$ (weak).

## References


[1] G. Müller, R. A. Richter, S. Weisbrod, and F. Klingberg, "Duration of EEG alpha wave blockade by tone stimulation is prolonged in early stage of presenile onset dementia of the Alzheimer type", *Biomed Biochim Acta*, vol. 50, no. 8, pp. 987–991, 1991.

[2] L. Defebvre, P. Derambure, J. L. Bourriez, J. M. Jacquesson, K. Dujardin, A. Destée, and J. D. Guieu, "Spatiotemporal study of event–related desynchronization in idiopathic Parkinson's disease", in *Advances in Neurology*, H. Narabayashi, T. Nagatsu, N. Yanagisawa, and Y. Mizuno, Eds., New York, 1993, pp. 422–428, Raven Press.

[3] J.P. Kaipio, *Simulation and Estimation of Nonstationary EEG*, PhD thesis, University of Kuopio, 1996.

[4] G. Pfurtscheller, "Event–related synchronization (ERS): an electrophysiological correlate of cortical areas at rest", *Electroencephalogr Clin Neurophysiol*, vol. 83, 1992.

[5] O.M. Markand, "Alpha rhythms", *J Clin Neurophysiol*, vol. 7, pp. 163–189, 1990.

[6] J.P. Kaipio and P.A. Karjalainen, "Estimation of event-related synchronization changes by a new TVARLS method", accepted to *IEEE Trans. Biomed. Eng.*, 1997.

[7] N. Amir and I. Gath, "Segmentation of EEG during sleep using time varying autoregressive modeling", *Biol Cybern*, vol. 61, pp. 447–455, 1989.

[8] J.P. Kaipio and P.A. Karjalainen, "Simulation of nonstationary EEG", accepted to *Biol. Cybern.*, 1997.

[9] C.L. Lawson and R.J. Hanson, *Solving Least Squares Problems*, SIAM, 1995.