Time-Varying Analysis of Heart Rate Variability with Kalman Smoother Algorithm

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***Abstract*— A time-varying parametric spectrum estimation method for analyzing nonstationary heart rate variability signals is presented. In the method, the nonstationary signal is first modeled with time-varying autoregressive model and the model parameters are estimated recursively with a Kalman smoother algorithm. The spectrum estimates for each time are then obtained from the estimated model parameters. Statistics of the obtained spectrum estimates are derived using the error propagation principle. The obtained spectrum estimates can further be decomposed into separate components and, thus, the time-variation of low and high frequency components of heart rate variability can be examined separately.**

I. Introduction

The variation of the heart beat interval or heart rate variability (HRV) is a consequence of the autonomic nervous systems control of the heart [1]. Both the sympathetic and parasympathetic branches are effective and as a result periodic components appear in HRV signal. The most conspicuous periodic component of HRV is the respiratory sinus arrhythmia (RSA) which is considered to range from 0.15 to 0.4 Hz. In addition to the physiological influence of breathing on HRV, this high frequency (HF) component is generally believed to be of parasympathetic origin. Another widely studied component of HRV is the low frequency (LF) component ranging form 0.04 to 0.15 Hz. The rhythms within the LF band are generally thought of being both of sympathetic and parasympathetic origin [1]. Thus, HRV is commonly examined through spectral analysis and, e.g., the LF/HF ratio is considered as an index of sympathovagal balance.

Due to the complex control systems of HRV, it is presumable that the characteristics of HRV vary in time. Especially, changes in physiological conditions may produce significant changes. For example, in the orthostatic test, subject stands up after lying supine for few minutes. After standing up, HR starts to increase to compensate the decrease in blood pressure. On supine, the high frequency variation of HR is typically strong, often higher than the low frequency variation. At the instant of standing, an immediate strong decrease in HF variation and a more gradual increase in LF variation has been observed [2]. In order to analyze such changes, time-frequency methods are required.

During the last decade, various such methods have been applied for HRV analysis. The applied time-frequency methods include short time Fourier transformation (STFT) and wavelet transform [2], [3], time-frequency distributions such as the Wigner distribution [4], [5], [6], [7], and time-varying autoregressive (AR) modeling based methods [8].

In this paper, we present a Kalman smoother method for estimating time-varying characteristics of HRV. In this method, the HRV signal is first modeled as an output of time-varying AR model. The time-varying model parameters are estimated recursively with a Kalman smoother algorithm. Also the error covariance for the AR parameter estimates is evaluated iteratively in the algorithm. The time-varying spectrum estimate is obtained from the estimated model parameters and its statistics can be evaluated by using the error propagation formula as derived in this paper. The time-varying spectrum can be further decomposed into separate spectral components which is especially advantageous in HRV applications, where LF and HF components are generally aimed to be distinguished.

II. Methods

The formulation of the Kalman smoother equations is based on *state-space formalism*. In this paper, the HRV dynamics are estimated using a time-varying AR model of order p given by

$$x_t = -\sum_{j=1}^{p} a_t^{(j)} x_{t-j} + e_t \tag{1}$$

where $a_t^{(j)}$ is the value of the j'th AR parameter at time t and e_t is the observation error. By denoting

$$H_t = (x_{t-1}, \ldots, x_{t-p}) \tag{2}$$

$$\theta_t = (-a_t^{(1)}, \ldots, -a_t^{(p)})^T \tag{3}$$

the time-varying AR model can be written in the form

$$x_t = H_t \theta_t + e_t \tag{4}$$

which is a linear observation model with H_t being the regression vector. The evolution of the state (i.e. the AR parameters) θ_t when no prior information is available is typically described with the random walk model

$$\theta_{t+1} = \theta_t + w_t \tag{5}$$

where w_t is the state noise term. Equations (4) and (5) form the state-space signal model for the time-varying AR process x_t and the evolution of the AR parameters can now be estimated using the Kalman smoother algorithm.

A. Kalman smoother algorithm

The Kalman smoother algorithm presented in this paper consists of a Kalman filter algorithm and a fixed-interval smoother. The Kalman filter is a real time processing algorithm in which the state estimate is updated immediately after a new observation is available. The fixed-interval smoother, on the other hand, estimates each state θ_t based on all the observations and, thus, the estimates are expected to be more accurate than the filter estimates.

The Kalman filtering problem is to find the linear mean square estimator $\hat{\theta}_t$ for state θ_t given the observations $x_1, x_2, \ldots, x_t$. Both the observation and state noises e_t and w_t are here assumed to be zero-mean random processes with covariances C_{e_t} and C_{w_t}. For the derivation of the Kalman filter equations see, e.g., [9]. For the time-varying AR model case, these equations can be written in the form

$$
\begin{aligned}
C_{\tilde{\theta}_{t|t-1}} &= C_{\tilde{\theta}_{t-1}} + C_{w_{t-1}} && (6)\\
K_t &= C_{\tilde{\theta}_{t|t-1}} H_t^T (H_t C_{\tilde{\theta}_{t|t-1}} H_t^T + C_{e_t})^{-1} && (7)\\
\hat{\theta}_t &= \hat{\theta}_{t-1} + K_t (x_t - H_t \hat{\theta}_{t-1}) && (8)\\
C_{\tilde{\theta}_t} &= (I - K_t H_t) C_{\tilde{\theta}_{t|t-1}} && (9)
\end{aligned}
$$

where $\tilde{\theta}_t$ is the state estimation error $\tilde{\theta}_t = \theta_t - \hat{\theta}_t$, $\tilde{\theta}_{t|t-1}$ is the state prediction error $\tilde{\theta}_{t|t-1} = \theta_t - \hat{\theta}_{t-1}$, and K_t is the Kalman gain vector.

The fixed-interval smoothing problem is to find estimates $\hat{\theta}_t^{\mathrm{S}}$ for each state θ_t given all the observations $x_1, x_2, \ldots, x_N$. The superscript S is used here to refer to smoothed estimates. The smoothing equations can be written in the form [10]

$$
\begin{aligned}
\hat{\theta}_t^{\mathrm{S}} &= \hat{\theta}_t + A_t (\hat{\theta}_{t+1}^{\mathrm{S}} - \hat{\theta}_t) && (10)\\
C_{\tilde{\theta}_t^{\mathrm{S}}} &= C_{\tilde{\theta}_t} + A_t (C_{\tilde{\theta}_{t+1}^{\mathrm{S}}} - C_{\tilde{\theta}_{t+1|t}}) A_t^T && (11)\\
A_t &= C_{\tilde{\theta}_t} C_{\tilde{\theta}_{t+1|t}}^{-1}. && (12)
\end{aligned}
$$

Thus, the smoothed estimates are obtained by running the filtered estimates backwards in time by taking $t = N-1, N-2, \ldots, 1$. The filtered estimates are used for the initialization, i.e. $\hat{\theta}_N^{\mathrm{S}} = \hat{\theta}_N$ and $C_{\tilde{\theta}_N^{\mathrm{S}}} = C_{\tilde{\theta}_N}$.

B. Initialization of the algorithm

To operate the Kalman filter algorithm, both the initial values for the state and its error covariance and the noise covariances C_{w_t} and C_{e_t} need to be specified. The smoother algorithm, on the other hand, does not require any additional specifications. A common approach for the initialization is to set the initial state, e.g., for $\hat{\theta}_0 = 0$ and its error covariance, e.g., for $C_{\tilde{\theta}_0} = I$ and then to run a short segment from the beginning of data backwards in time. The state noise covariance C_{w_t} and the observation noise covariance C_{e_t} are the terms that determine the adaptation (i.e. the speed of change of the state) of the Kalman filter through the Kalman gain vector K_t. Here we have estimated the observation noise variance iteratively in every step of the Kalman filter equations as

$$\hat{\sigma}_{e_t}^2 = 0.95\, \hat{\sigma}_{e_{t-1}}^2 + 0.05\, \epsilon_t^2 \tag{13}$$

where ϵ_t is the one step prediction error $\epsilon_t = x_t - H_t \hat{\theta}_{t-1}$. The state noise covariance, on the other hand, is assumed diagonal and the covariance coefficient σ_w^2 is adjusted in every step of the Kalman filter equations as

$$\hat{\sigma}_{w_t}^2 = \mathrm{UC}\, \hat{\sigma}_{e_t}^2 I \tag{14}$$

where UC is an update coefficient through which the adaptation of the algorithm is adjusted. The bigger the value of UC the quicker the adaptation. The variance of the state estimates is, however, inversely proportional to the value of UC and, therefore, UC should be specified in such a way that a desired balance between the filter adaptation and estimate variance is obtained.

C. Time-varying spectrum estimation

The time-varying spectrum estimate is obtained from the momentary AR parameter estimates $\hat{a}_t^{(j)}$ as

$$P_t(f) = \frac{\hat{\sigma}_e^2 / f_s}{|1 + \sum_{j=1}^{p} \hat{a}_t^{(j)} e^{-i2\pi j f / f_s}|^2} \tag{15}$$

where f_s is the sampling frequency, $\hat{a}_t^{(j)}$ is the AR parameter estimate at time t, and $\hat{\sigma}_e^2$ is the variance of the observation error process.

D. Statistics of Kalman smoother spectrum estimates

The covariance matrix of the state estimation error $\tilde{\theta}_t$ is calculated iteratively at every step of the Kalman smoother algorithm in (11). Thus the variance and covariance of AR parameter estimates at each time t are known and the variance of the spectrum estimate at time t can be evaluated by using the error propagation formula

$$
\begin{aligned}
\sigma_{P_t(f)}^2 &= \sum_{k=1}^{p} \left(\frac{\partial P_t(f)}{\partial \hat{a}_t^{(k)}} \right)^2 \sigma_{\hat{a}_t^{(k)}}^2 \\
&+ \sum_{k=1}^{p} \sum_{\substack{l=1 \\ l \neq k}}^{p} \frac{\partial P_t(f)}{\partial \hat{a}_t^{(k)}} \frac{\partial P_t(f)}{\partial \hat{a}_t^{(l)}} \sigma_{\hat{a}_t^{(k)} \hat{a}_t^{(l)}} && (16)
\end{aligned}
$$

where $\sigma_{\hat{a}_t^{(k)}}^2$ is the variance of the k'th AR parameter estimate, $\sigma_{\hat{a}_t^{(k)} \hat{a}_t^{(l)}}$ is the covariance of k'th and l'th AR parameter estimates, and the partial derivative of $P_t(f)$ with respect to $\hat{a}_t^{(k)}$ can be written in the form

$$\frac{\partial P_t(f)}{\partial \hat{a}_t^{(k)}} = \frac{-2\hat{\sigma}_e^2 / f_s \left(\cos\omega_k + \sum_{j=1}^{p} \hat{a}_t^{(j)} \cos(\omega_j - \omega_k) \right)}{\left[(1 + \sum_{j=1}^{p} \hat{a}_t^{(j)} \cos\omega_j)^2 + (\sum_{j=1}^{p} \hat{a}_t^{(j)} \sin\omega_j)^2 \right]^2} \tag{17}$$

where $\omega_j = 2\pi j f / f_s$. Further derivation of, e.g., specific band power variances using the error propagation formula is straightforward.

Note that we did not include any error term for the frequency f in the error propagation formula. The frequency resolution of the AR spectrum is naturally not, however, infinite, but it is determined by the underlying model. The frequency resolution of parametric spectrum estimation methods is, nevertheless, higher than that of FFT based methods and, thus, its influence on the variance of the spectrum estimate can be assumed to be of minor significance.

E. Spectral decomposition

One property of the AR spectrum estimation methods, that is especially advantageous in HRV applications, is that the spectrum can be divided into separate components as follows. Equation (15) can also be written in the factored form

$$P_t(f) = \frac{\hat{\sigma}_e^2/f_s}{\prod_{j=1}^{p}(z-\alpha_t^{(j)})(1/z-\alpha_t^{(j)*})} \qquad (18)$$

where $z = e^{i2\pi f/f_s}$, $\alpha_t^{(j)}$ are the time-varying roots of the AR polynomial (also called poles), and * denotes complex conjugate. Now, consider a pole $\alpha_t^{(j)}$ positioned at frequency f_j. The spectrum of this single component in the vicinity of f_j can be estimated as

$$P_t^{(j)}(f) \approx \frac{c_t^{(j)}}{(z-\alpha_t^{(j)})(1/z-\alpha_t^{(j)*})}, \qquad z = e^{i2\pi f/f_s} \qquad (19)$$

where the constant $c_t^{(j)}$ is given by

$$c_t^{(j)} \approx \frac{\hat{\sigma}_e^2/f_s}{\prod_{\substack{k=1\\k\neq j}}^{p}(z-\alpha_t^{(k)})(1/z-\alpha_t^{(k)*})}, \qquad z = e^{i2\pi f_j/f_s}. \qquad (20)$$

That is, the part $c_t^{(j)}$ of the AR spectrum estimate is assumed to be constant when $f \approx f_j$. The sum of the component spectra is approximately equal to the AR spectrum estimate, i.e. $P_t(f) \approx \sum_{j=1}^{p} P_t^{(j)}(f)$.

The powers of the spectral components can be estimated, e.g., by using the method proposed in [11]. In this approach, the power of the component positioned at frequency f_j is estimated with the residue

$$P_t^{f_j} = d\,\mathrm{Re}\left\{\mathrm{Res}\left\{\frac{P_t(z)}{z}\right\}\Big|_{z=e^{i2\pi f_j/f_s}}\right\} \qquad (21)$$

where the residue is evaluated at $z = e^{i2\pi f_j/f_s}$ and the coefficient $d = 1$ for real poles and $d = 2$ for complex poles. The previous equation can be solved by evaluating

$$P_t^{f_j} = d\,\mathrm{Re}\left\{\frac{\hat{\sigma}_e^2(z-\alpha_t^{(j)})}{zA(z)A(1/z)}\right\} \qquad (22)$$

where $A(z) = \prod_{k=1}^{p}(z-\alpha_t^{(k)})$ and $A(1/z) = \prod_{k=1}^{p}(1/z-\alpha_t^{(k)*})$, at $z = \alpha_t^{(j)}$. This method for component power estimation works for well-separated poles, but for poles close to each other power estimates can yield even negative values. A more robust way to estimate the powers is, however, to simply calculate the areas of the spectral components.

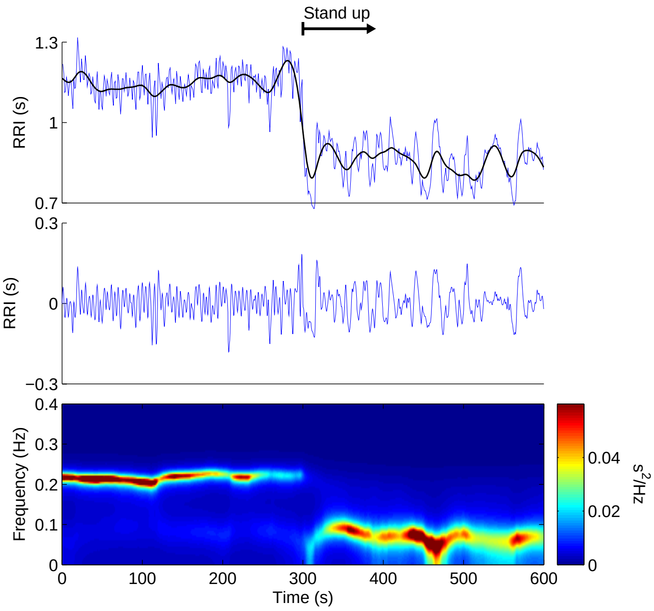

Fig. 1. Time-varying spectrum estimation of HRV during an orthostatic test. The measured RR interval series and the estimated trend (top), detrended RR interval series (middle), and the Kalman smoother spectrum estimate (bottom).

III. Results

As a case study, we examined the dynamics of HRV during an orthostatic test. In the test, the subject (a healthy young male) lay supine for over five minutes and then stood up for another five minutes. The ECG signal was measured using a Compumedics Neuroscan measurement system (sampling frequency 500 Hz). The R-wave positions were extracted with an adaptive QRS detection algorithm and the RR interval series was formed. The RR interval series was further transformed to evenly sampled time series by using a 4 Hz cubic spline interpolation.

The obtained RR interval series is presented on top of Fig. 1. The low frequency trend components of HRV can distort the AR spectrum estimate and, thus, the trend was removed by using a smoothness priors based method described in [12]. The detrended RR interval series and its Kalman smoother spectrum estimate are also presented in Fig. 1. The update coefficient UC of the Kalman smoother algorithm was set to 0.05. The stand up instant is at 300 seconds and the changes in the powers of HF and LF components due to this posture change are evident.

The LF and HF band powers and the LF/HF ratio as a function of time with 95 % confidence intervals are presented in Fig. 2. The band powers are simply obtained by integrating the spectrum over the specific frequency band. The variances of the band powers were calculated by using the error propagation formalism described in Section II-D.

The decomposition of the Kalman smoother spectrum into LF and HF spectral components is shown in Fig. 3. The decomposition of the spectra at $t = 150$ and $t = 400$

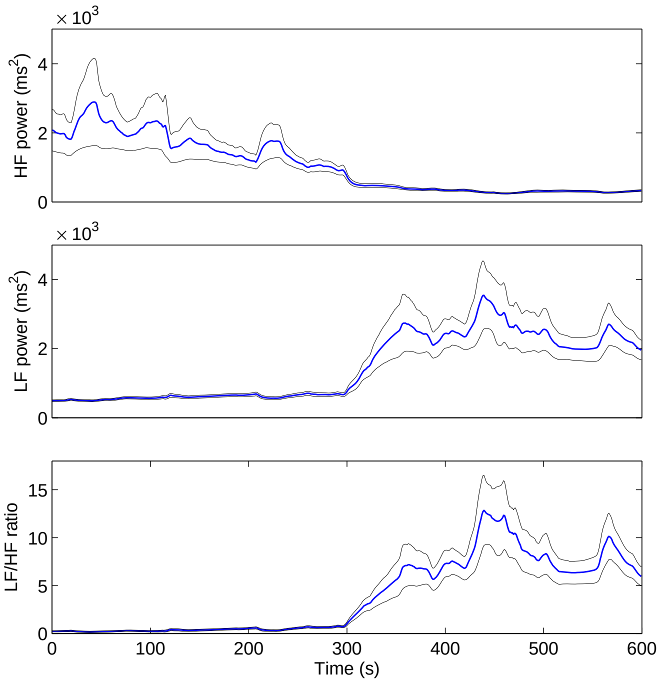

Fig. 2. Estimated dynamics of LF and HF band powers and LF/HF ratio with 95% confidence intervals.

seconds is illustrated on top of the figure. By using the spectral decomposition separate spectral estimates can be formed for the different frequency components.

IV. Discussion

A Kalman smoother spectrum estimation method for time-varying estimation of HRV dynamics was presented. Kalman smoother is an optimal linear mean square estimator of the AR parameters and, thus, the method is statistically favorable. In addition, the frequency resolution of AR spectrum estimates is better than that of FFT based spectrum estimates due to implicit extrapolation of the autocorrelation. Thus, the time-frequency resolution of the Kalman smoother spectrum is highly competent.

The adaptation of the proposed spectrum estimation method can be adjusted with a single parameter, i.e. the update coefficient UC. The decision of this coefficient should be done by compromising between the adaptation speed and the variance of the AR parameter estimates. Using the error propagation formula presented in this paper, the variance of the corresponding AR spectrum estimate can be further evaluated.

One advantage of the presented method is that the spectrum can be divided into separate frequency components. Thereby, it enables the separate estimation of the LF and HF frequency components. In addition, in case of overlapping spectral components, it is expectable that the component powers can be evaluated more accurately from the decomposed spectra.

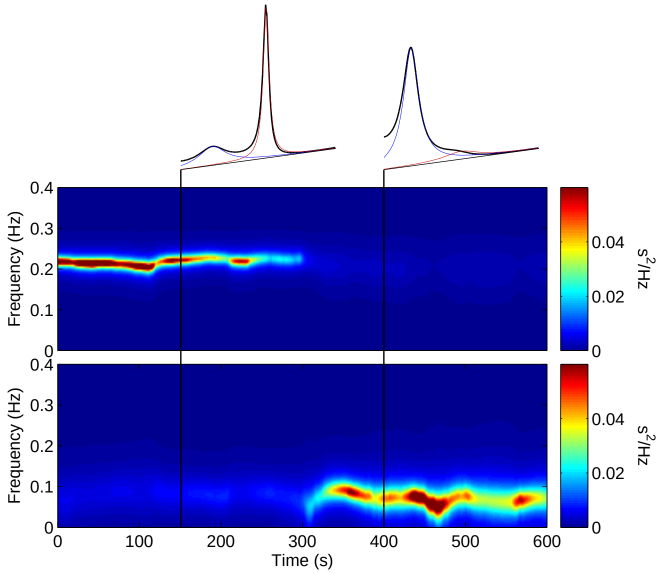

Fig. 3. Decomposition of the Kalman smoother spectrum into LF (bottom) and HF (top) spectral components.

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