TIME-VARYING ANALYSIS OF HEART RATE VARIABILITY WITH KALMAN SMOOTHER ALGORITHM

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Abstract

A time-varying parametric spectrum estimation method for analyzing nonstationary heart rate variability (HRV) signals is presented. The presented method is based on the Kalman smoother algorithm and yields high resolution spectral estimates for analyzing the dynamics of HRV.

Introduction

- Heart rate variability (HRV) is a consequence of the autonomic nervous systems control of the heart.
- Due to complex control systems of HRV, it is presumable that the characteristics of HRV vary in time.
- Various time-frequency methods have been applied for HRV analysis. These include short time Fourier transform (STFT) and wavelet transform, time-frequency distributions such as the Wigner distribution, and time-varying autoregressive (AR) modeling based methods.
- We present a Kalman smoother approach for estimating HRV dynamics. In this method, the HRV signal is first modeled with time-varying AR model. The model parameters are then estimated recursively with the Kalman smoother algorithm. The time-varying spectrum estimate is obtained from the estimated model parameters and its statistics can be evaluated by using the error propagation principle. The obtained spectrum estimate can further be decomposed into separate, e.g. LF and HF, components.

Methods

State-space formalism

The formulation of Kalman smoother equations is based on *state-space formalism.*

- Here, the HRV dynamics are estimated using a time-varying AR model

$$x_t = -\sum_{j=1}^{p} a_t^{(j)} x_{t-j} + e_t$$

where $a_t^{(j)}$ is the j'th AR parameter at time t and e_t is observation error. The state-space equations for this can be written as

$$\begin{aligned} x_t &= H_t \theta_t + e_t \\ \theta_{t+1} &= \theta_t + w_t \end{aligned}$$

where $H_t = (x_{t-1}, \ldots, x_{t-p})$, $\theta_t = (-a_t^{(1)}, \ldots, -a_t^{(p)})^T$, and w_t is the state noise term.

Kalman smoother algorithm

The Kalman smoother algorithm consists of two steps:
1) Forward run of data with Kalman filter algorithm
2) Backward run with fixed-interval smoother algorithm.

- The Kalman filtering problem is to find the linear MS estimator $\hat{\theta}_t$ for state θ_t given the observations $x_1, x_2, \ldots, x_t$. Kalman filter equations can be written in the form [1, 2]

$$\begin{aligned} C_{\tilde{\theta}_{t|t-1}} &= C_{\tilde{\theta}_{t-1}} + C_{w_{t-1}} \\ K_t &= C_{\tilde{\theta}_{t|t-1}} H_t^T (H_t C_{\tilde{\theta}_{t|t-1}} H_t^T + C_{e_t})^{-1} \\ \hat{\theta}_t &= \hat{\theta}_{t-1} + K_t (x_t - H_t \hat{\theta}_{t-1}) \\ C_{\tilde{\theta}_t} &= (I - K_t H_t) C_{\tilde{\theta}_{t|t-1}} \end{aligned}$$

where $\tilde{\theta}_t$ is the state estimation error $\tilde{\theta}_t = \theta_t - \hat{\theta}_t$, $\tilde{\theta}_{t|t-1}$ is the state prediction error $\tilde{\theta}_{t|t-1} = \theta_t - \hat{\theta}_{t-1}$, K_t is the Kalman gain vector, and C denotes covariance matrices.

- The fixed-interval smoothing problem is to find estimates $\hat{\theta}^S{}_t$ for each state θ_t given all the observations $x_1, x_2, \ldots, x_N$. The smoothing equations can be written in the form [2]

$$\begin{aligned} \hat{\theta}^S{}_t &= \hat{\theta}_t + A_t (\hat{\theta}^S{}_{t+1} - \hat{\theta}_t) \\ C_{\tilde{\theta}^S_t} &= C_{\tilde{\theta}_t} + A_t (C_{\tilde{\theta}^S_{t+1}} - C_{\tilde{\theta}_{t+1|t}}) A_t^T \\ A_t &= C_{\tilde{\theta}_t} C^{-1}_{\tilde{\theta}_{t+1|t}} \end{aligned}$$

and the smoothed estimates are obtained by running the filtered estimates backwards in time by taking $t = N-1, N-2, \ldots, 1$.

Initialization

- The state estimate $\hat{\theta}_t$ and its error covariance $C_{\tilde{\theta}_t}$ are initialized by running a short segment from the beginning of data backwards in time.
- Observation covariance $C_{e_t} = \sigma^2_{e_t}$ is estimated iteratively as

$$\hat{\sigma}^2_{e_t} = 0.95\, \hat{\sigma}^2_{e_{t-1}} + 0.05\, \epsilon_t^2$$

where ϵ_t is the one-step prediction error $\epsilon_t = x_t - H_t \hat{\theta}_{t-1}$.

- State covariance $C_{w_t} = \sigma^2_{w_t} I$ is adjusted as

$$\hat{\sigma}^2_{w_t} = \mathrm{UC}\, \hat{\sigma}^2_{e_t} / \hat{\sigma}^2_{x_t} I$$

where $\hat{\sigma}^2_{x_t}$ is the variance of the HRV signal at time t and UC is an update coefficient through which the adaptation of the algorithm can be adjusted.

Time-varying spectrum estimation

The time-varying spectrum estimate is obtained from the momentary AR parameter estimates $\hat{a}_t^{(j)}$ as

$$P_t(f) = \frac{\hat{\sigma}_e^2 / f_s}{|1 + \sum_{j=1}^{p} \hat{a}_t^{(j)} e^{-i 2\pi j f / f_s}|^2}$$

where f_s is the sampling frequency and $\hat{\sigma}_e^2$ is the variance of the estimated observation error process.

Statistics of the spectrum estimates

The error covariance of the AR parameters $C_{\tilde{\theta}_t}$ is estimated iteratively in the Kalman smoother algorithm ➠ Variance of the spectrum estimate can be evaluated by using the error propagation formula

$$\sigma^2_{P_t(f)} = \sum_{k=1}^{p} \left(\frac{\partial P_t(f)}{\partial \hat{a}_t^{(k)}} \right)^2 \sigma^2_{\hat{a}_t^{(k)}} + \sum_{k=1}^{p} \sum_{\substack{l=1 \\ l \neq k}}^{p} \frac{\partial P_t(f)}{\partial \hat{a}_t^{(k)}} \frac{\partial P_t(f)}{\partial \hat{a}_t^{(l)}} \sigma_{\hat{a}_t^{(k)} \hat{a}_t^{(l)}}$$

where $\sigma^2_{\hat{a}_t^{(k)}}$ is the error variance of the k'th AR parameter estimate and $\sigma_{\hat{a}_t^{(k)} \hat{a}_t^{(l)}}$ is the error covariance of k'th and l'th AR parameter estimates.

Spectral decomposition

AR spectrum can be divided into separate frequency components as follows.

- AR spectrum equation can be written in the factored form

$$P_t(f) = \frac{\hat{\sigma}_e^2 / f_s}{\prod_{j=1}^{p} (z - \alpha_t^{(j)})(1/z - \alpha_t^{(j)*})}$$

where $z = e^{i 2\pi f / f_s}$ and $\alpha_t^{(j)}$ are the time-varying roots of the AR polynomial.

- Consider a pole $\alpha_t^{(j)}$ positioned at frequency f_j. The spectrum of this single component in the vicinity of f_j can be estimated as

$$P_t^{(j)}(f) \approx \frac{c_t^{(j)}}{(z - \alpha_t^{(j)})(1/z - \alpha_t^{(j)*})}, \quad z = e^{i 2\pi f / f_s}$$

where the constant $c_t^{(j)}$ is given by

$$c_t^{(j)} \approx \frac{\hat{\sigma}_e^2 / f_s}{\prod_{\substack{k=1 \\ k \neq j}}^{p} (z - \alpha_t^{(k)})(1/z - \alpha_t^{(k)*})}, \quad z = e^{i 2\pi f_j / f_s}.$$

- Powers of the components can be estimated, e.g., by using a residue method proposed in [3] or simply by integrating the components.

Results

- As a case study, the dynamics of HRV during an orthostatic test were examined, see Fig. 1.
- The trend within the RR series was removed by using a smoothness priors based method described in [4].
- Kalman smoother spectrum estimate was calculated by using AR model order $p = 16$ and update coefficient $\mathrm{UC} = 1 \cdot 10^{-5}$.

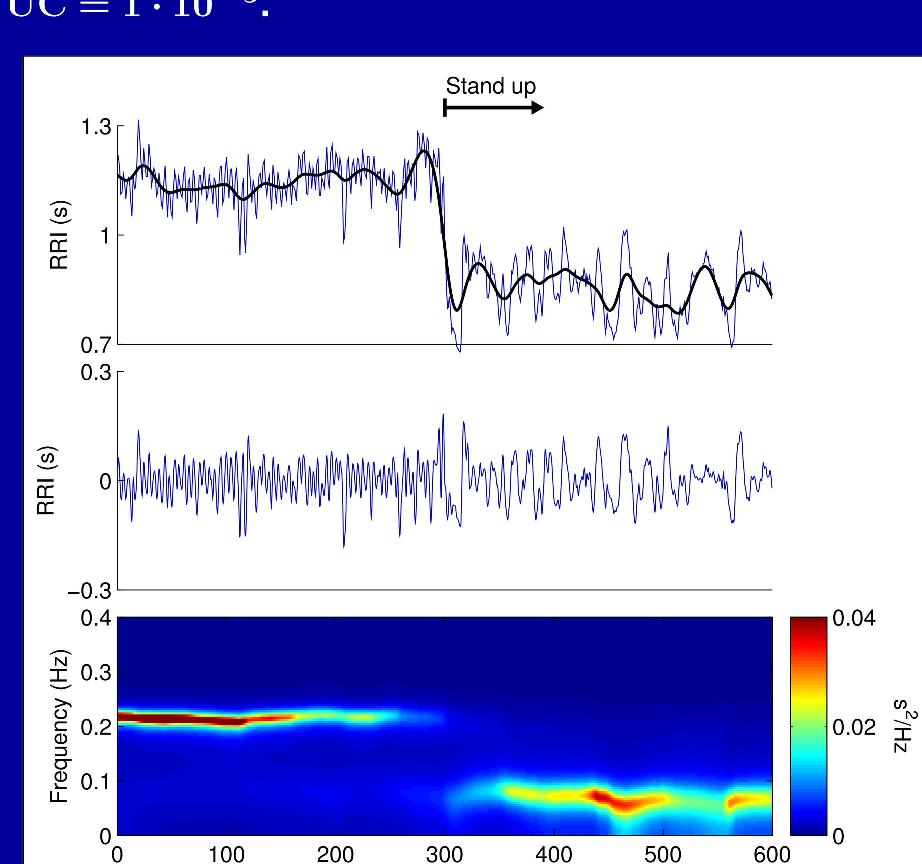

Fig. 1. Kalman smoother spectrum estimate of HRV series measured during an orthostatic test.

- The variance of the Kalman smoother spectrum was evaluated by using the error propagation formula. Spectrum estimates with $\pm 2\mathrm{SD}$ intervals at $t = 200$ and $t = 400$ seconds are presented in Fig. 2.

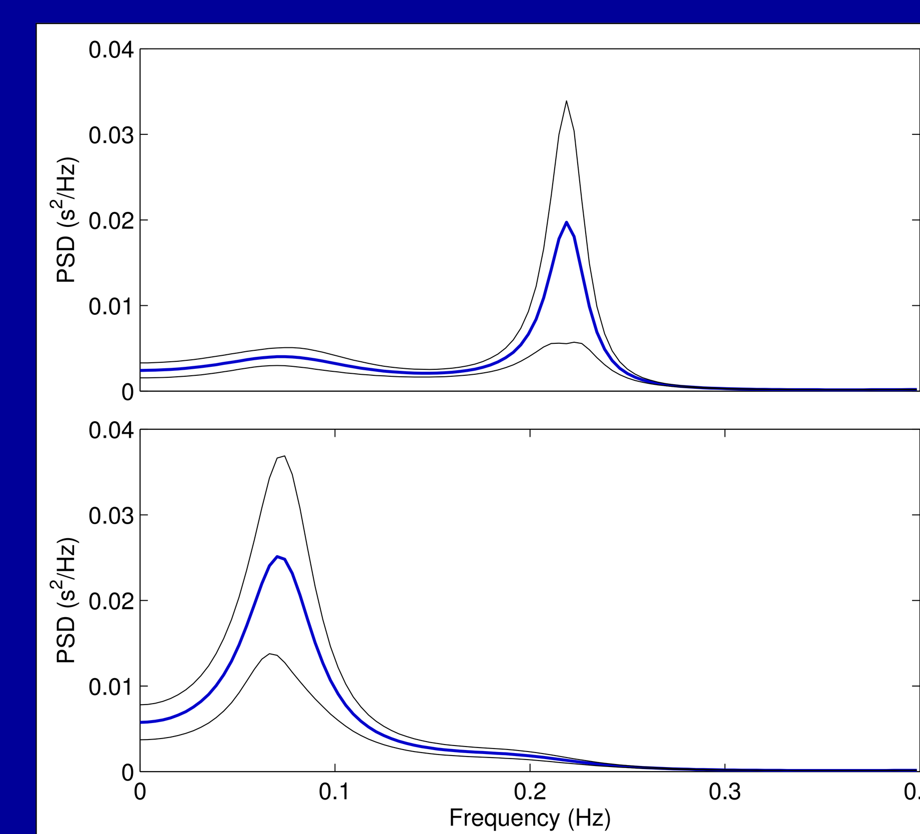

Fig. 2. Statistics of Kalman smoother spectrum estimates.

- LF and HF band powers and LF/HF ratio were then calculated. The results with $\pm 2\mathrm{SD}$ intervals are presented in Fig. 3.

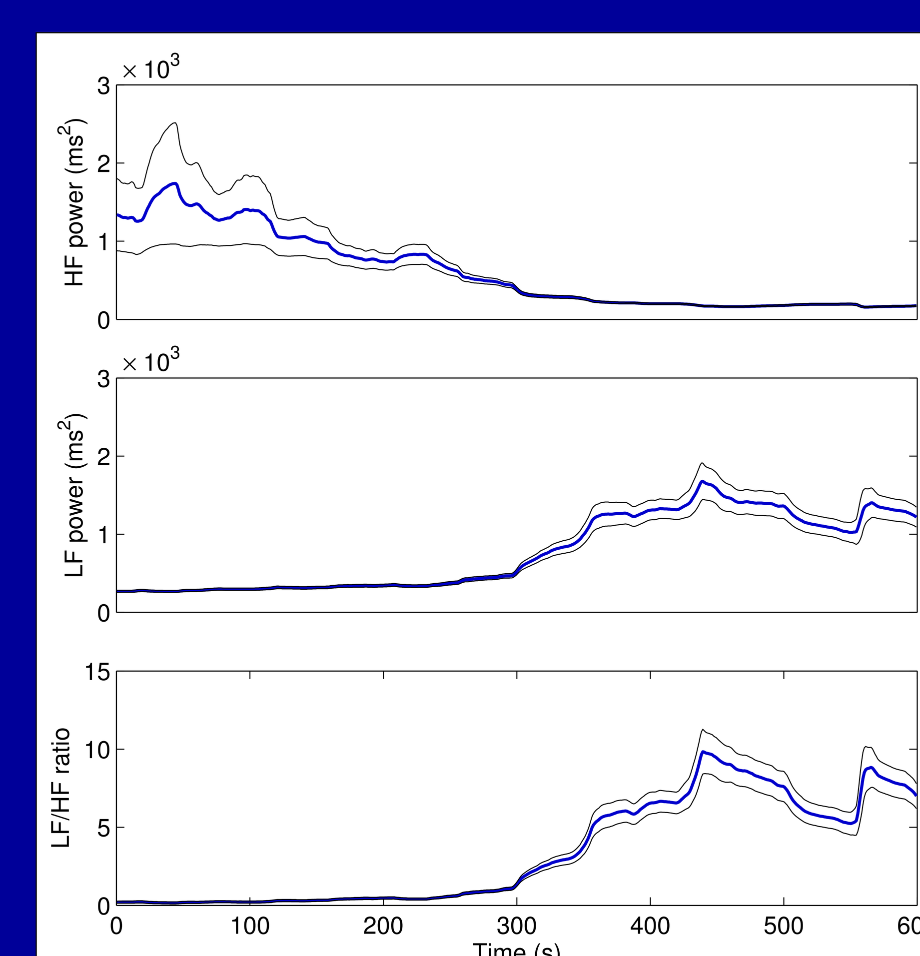

Fig. 3. Dynamics of LF and HF band powers and LF/HF ratio.

- Finally, the decomposition of the Kalman smoother spectrum into LF and HF components is shown in Fig. 4.

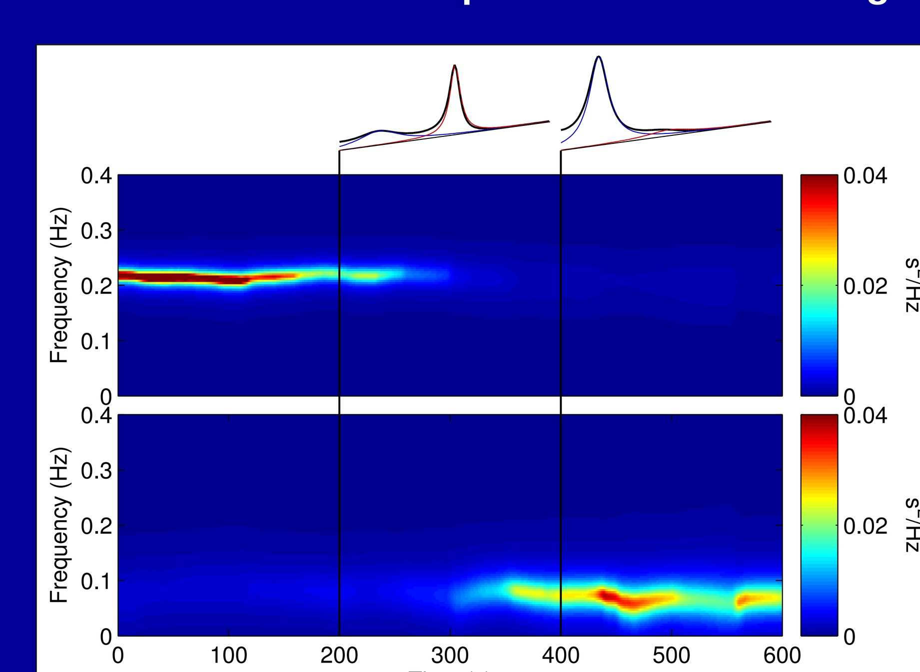

Fig. 4. Decomposition of the Kalman smoother spectrum.

Conclusions

- The Kalman smoother approach yields high-resolution spectrum estimates for studying HRV dynamics.
- Adaptation of the method can be adjusted with a single parameter, i.e. the update coefficient UC.
- Statistics of the spectrum estimate can be evaluated by using the error propagation formula.
- Spectrum can be divided into separate components, enabling the separate estimation of LF and HF frequency components.

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