TRACKING OF NONSTATIONARY EEG WITH KALMAN SMOOTHER APPROACH: AN APPLICATION TO EVENT-RELATED SYNCHRONIZATION OF ALPHA WAVES

M.P. Tarvainen[1], J.K. Hiltunen[2], P.O. Ranta-aho[1], and P.A. Karjalainen[1]
[1]Department of Applied Physics, University of Kuopio, Kuopio, Finland
[2]Brain Work Laboratory, Finnish Institute of Occupational Health, Helsinki, Finland

Abstract— **An adaptive spectrum estimation method for nonstationary EEG by means of Kalman filtering along with fixed-interval smoothing is presented. The advantages of the Kalman smoother approach are the optimality properties and the avoidance of the tracking lag present in all adaptive algorithms. The presented Kalman smoother approach was applied to tracking of event-related synchronization of EEG and high resolution estimates for EEG in alpha frequency band were obtained.**

Keywords— **Nonstationary EEG, adaptive algorithms, Kalman smoother**

I. Introduction

The electroencephalogram (EEG) recording is a useful tool for studying the functional states of the brain and for diagnosing some neurophysiological pathologies. For nonstationary EEG signals time-frequency representations enable the time-varying spectral analysis. A traditional time-frequency method is the moving window Fourier transformation which is often called the spectrogram. Disadvantages of this method are the implicit assumption of stationarity within each windowed segment and the trade-off between time and frequency resolutions.

A better approach is to use parametric spectral estimation methods based on e.g. time-varying autoregressive moving average (ARMA) modeling. Adaptive algorithms such as the Kalman filter can be used to solve the time-varying parameter estimation problem [1], [2]. The tracking lag of all adaptive algorithms can further be avoided by using a so-called smoother along with the Kalman filter [1]. In this paper we apply the Kalman smoother algorithm in the spectral estimation of an event-related desynchronization/synchronization (ERD/ERS) of EEG. Due to the optimality properties of the Kalman smoother accurate estimates for single ERD/ERS epochs are obtained.

II. Methodology

A. Kalman smoother approach

The time-varying ARMA(p,q) model for signal z_t at time instant t can be written in the form

$$z_t = -\sum_{j=1}^{p} a_t^{(j)} z_{t-j} + \sum_{k=1}^{q} b_t^{(k)} e_{t-k} + e_t \qquad (1)$$

where $a_t^{(j)}$ and $b_t^{(k)}$ are the time-varying AR and MA parameters, p and q are the model orders, and e_t is a white noise process. By denoting

$$\theta_t = \left(-a_t^{(1)}, \ldots, -a_t^{(p)}, b_t^{(1)}, \ldots, b_t^{(q)}\right)^T \qquad (2)$$

$$\varphi_t = \left(z_{t-1}, \ldots, z_{t-p}, e_{t-1}, \ldots, e_{t-q}\right)^T \qquad (3)$$

the ARMA model can be written in the form

$$z_t = \varphi_t^T \theta_t + e_t, \quad \text{(space equation)}. \qquad (4)$$

The state evolution when no *a priori* information is available is usually described by the random walk model [2]

$$\theta_{t+1} = \theta_t + w_t, \quad \text{(state equation)} \qquad (5)$$

where w_t is a white noise process. The time-varying parameters θ_t can now be solved with the Kalman filter.

The Kalman filtering problem is to find the minimum mean square estimator $\hat{\theta}_t$ for state θ_t given the observations $z_1, \ldots, z_t$. With certain assumptions Kalman filter equations can be written in the form [3], [1], [2]

$$C_{\tilde{\theta}_{t|t-1}} = C_{\tilde{\theta}_{t-1}} + C_{w_{t-1}} \qquad (6)$$

$$K_t = C_{\tilde{\theta}_{t|t-1}} \varphi_t \left(\varphi_t^T C_{\tilde{\theta}_{t|t-1}} \varphi_t + C_{e_t}\right)^{-1} \qquad (7)$$

$$C_{\tilde{\theta}_t} = \left(I - K_t \varphi_t^T\right) C_{\tilde{\theta}_{t|t-1}} \qquad (8)$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} + K_t \left(z_t - \varphi_t^T \hat{\theta}_{t-1}\right) \qquad (9)$$

where $\tilde{\theta}_t = \theta_t - \hat{\theta}_t$ is the estimation error, K_t is the Kalman gain vector, and C denotes covariance matrices.

In the fixed-interval smoothing the smoothed estimates $\hat{\theta}_{t|T}$ for fixed T and for all t in the interval $[1, T]$ are obtained by running the stored Kalman filter estimates backwards in time [1]

$$\hat{\theta}_{t-1|T} = \hat{\theta}_{t-1} + A_{t-1}\left(\hat{\theta}_{t|T} - \hat{\theta}_{t-1}\right) \qquad (10)$$

$$A_{t-1} = C_{\tilde{\theta}_{t-1}} C_{\tilde{\theta}_{t|t-1}}^{-1}. \qquad (11)$$

B. Spectral estimation

Once the time-varying coefficients of the ARMA(p,q) model (1) are solved the time-varying power spectrum estimate at each time instant can be obtained in terms of the estimated ARMA coefficients

$$P_t(\omega) = \sigma_e^2(t) \frac{|1 + \sum_{k=1}^{q} b_t^{(k)} e^{-i\omega k}|^2}{|1 + \sum_{j=1}^{p} a_t^{(j)} e^{-i\omega j}|^2} \qquad (12)$$

where $\sigma_e^2(t)$ is the prediction error variance.

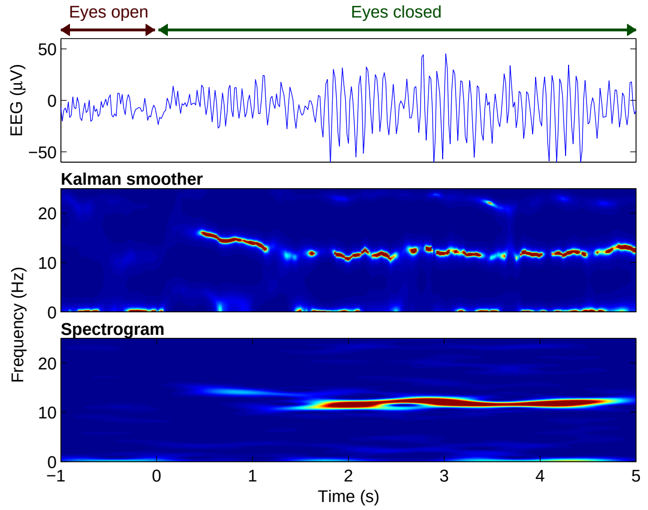

Fig. 1. Time-varying spectral estimation of an event-related alpha wave synchronization. The measured EEG from channel O2 is presented on the top and the spectrum estimates with Kalman smoother and spectrogram methods underneath it.

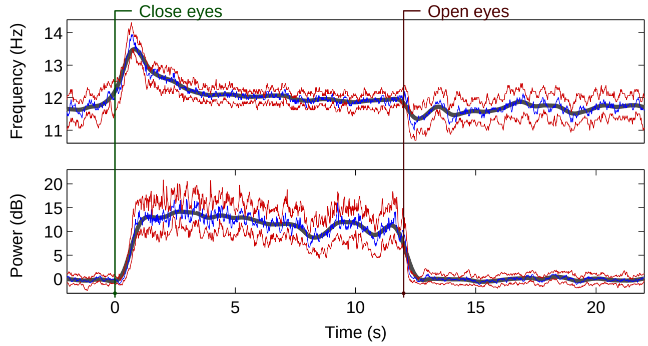

Fig. 2. Variation of the center frequency and peak power of alpha band for an eyes open/closed test; averages over 57 trials (—), 30% confidence limits (—), and estimated trends (▬).

III. Results

The event-related desynchronization/synchronization (ERD/ERS) in occipital brain region was tested by opening and closing the eyes with auditory stimuli at 12 second intervals. With this experiment, the ERD/ERS is best seen in the posterior and occipital regions of the right hemisphere and therefore, channel O2 was selected for analysis. The sampling frequency of the signal was decimated after low-pass filtering from 256 Hz to 64 Hz. A typical transition from desynchronized state to synchronized state obtained for the test subject (healthy young female) is presented in Fig. 1.

The time-varying spectrum estimate with the Kalman smoother as compared to the spectrogram is presented in Fig. 1. An ARMA(6,2) model was used in the Kalman smoother approach. In the spectrogram method a trade-off between the time and frequency resolutions is committed when selecting the width of the sliding time-window. Here we used a 1.5-second time-window. The observed resolution of the Kalman smoother spectrum is very accurate. Even the short-term changes in alpha activity (e.g. the attenuation of alpha waves in the interval 1.2–1.6 seconds) are detected reliably.

Furthermore, after eye closure the synchronization starts at higher frequency and gradually shifts to lower frequencies. This is a so-called "squeak" phenomenon of EEG [4], [5] and can not be seen from the spectrogram due to poor spectral resolution. Kalman smoother approach is, however, accurate enough to reveal the "squeak" effect even from a single ERD/ERS epoch. Fig. 2 shows the averaged Kalman smoother estimates over 57 eyes closed/open epochs and the estimated trends of the center frequency and peak power of the alpha waves. Since the characteristic alpha frequency for the test subject was as high as 12 Hz the band limits for synchronization were set to 8–16 Hz. The center frequency was defined as the frequency bisecting the power of this band.

IV. Discussion

We have applied the Kalman filtering algorithm along with a fixed-interval smoother to tracking of nonstationary EEG. The Kalman filter has been used in EEG analysis in e.g. [6], [7]. The Kalman smoother, on the other hand, has not been previously used for EEG. Our motivation for using such a complex and computationally demanding algorithm for EEG analysis was the optimality properties of the algorithm. Another advantage of the Kalman smoother is the avoidance of the tracking lag present with other adaptive algorithms. For these reasons Kalman smoother spectra have high time-frequency resolution and enable accurate modeling of some short-term EEG transients or event-related changes like the ERD/ERS of alpha waves. As accurate results for the ERD/ERS can not be obtained with the popular spectrogram method in epoch by epoch manner but only by averaging several recurred epochs. The disadvantage of the averaging is, however, that the ERD/ERS epochs are assumed to be alike and therefore, the information of single epoch variations is naturally lost.

References

[1] B. Anderson and J. Moore, *Optimal Filtering*. Prentice Hall, 1979.

[2] S. Haykin, *Adaptive Filter Theory*. Prentice-Hall, 1986.

[3] P. Karjalainen, *Regularization and Bayesian methods for evoked potential esimation*. PhD thesis, University of Kuopio, Department of Applied Physics, 1997.

[4] O. Markand, "Alpha rhythms," *J Clin Neurophysiol*, vol. 7, pp. 163–189, 1990.

[5] N. Kawabata, "A nonstationary analysis of the electroencephalogram," *IEEE Trans Biomed Eng*, vol. 20, pp. 444–452, 1973.

[6] T. Bohlin, "Analysis of EEG signals with changing spectra using a short-word Kalman estimator," *Math Biosci*, vol. 35, pp. 221–259, 1977.

[7] A. Isaksson, A. Wennberg, and L. Zetterberg, "Computer analysis of EEG signals with parametric models," *Proc IEEE*, vol. 69, pp. 451–461, 1981.