Single Trial Analysis of Event Related Potentials – Sensitivity of the Subspace Regularization Based Methods to the Value of Regularization Parameter

P.O. Ranta-aho[1], M.T. Tarvainen[1], A.S. Holm[2], P.A. Karjalainen[1]
[1] University of Kuopio, Department of Applied Physics, Kuopio, Finland
[2] Finnish Institute of Occupational Health, Brain@Work Laboratory, Helsinki, Finland

Abstract— **Effect of the regularization parameter to the single-trial estimates obtained with two subspace regularization based estimation methods is studied. First method relies on the single channel data. Second method is extension to multi channel case. It takes the spatial correlation of the channels in to account in the estimation.**

Methods are compared using simulated data. Simulations are based on combination of real measured background EEG and simulated scalp potentials caused by two time-varying dipoles.

Both methods are shown to be quite robust to non optimal selection of the regularization parameter. It is also shown that the ability of the multi channel method to preserve amplitude differences between channels is not due to selection of the regularization parameter.

Keywords— **ERP, subspace regularization, single trial estimation**

I. Introduction

Event related potentials (ERPs) are used for studying sensory and cognitive processing of the brain. Variation of the responses from stimuli to stimuli carriers valuable information to be used for example in the analysis of mental work load.

The methods for single trial analysis of ERPs are usually based on filtering of the data, time frequency analysis of data, or statistical analysis of ERP time series. Usually these methods are hard to extend to take into account the spatial correlation of the measurement channels. This makes comparison of the obtained peak amplitude estimates across channels difficult.

An interesting way to take this coupling of the measurement channel into account in the estimation is to use subspace regularization based method [1], [2] The main benefit of the methods is that they are applicable for different kind of data and that the multi channel method does not necessitate electrical modelling of the head. Instead the coupling of the channels is taken into account in the form of a priori information. The multi channel method relies only on the statistical properties of the data and does not necessitate information about the biophysics or geometry of the head.

However, as with all regularization based algorithm the performance of the methods depends on the selection of the regularization parameter. Here we will study the effect of the regularization parameter to the peak amplitude estimates obtained with subspace regularization based method.

II. Linear model

We use commonly used linear model for evoked potentials. That is, we model the individual measurement z_i as summation of event related activity s_i and non event related (background) activity e_i. Furthermore the event related part s_i is modeled as linear combination of some basis functions. We can write our observation model in form

$$z = s + e = H\theta + e \tag{1}$$

where θ are the parameters and matrix H contains the basis functions in its columns. z and s are matrices containing data of each individual measurement in their columns.

This kind of model relies on selection of the basis functions. Generally the choice of the basis functions implies some assumptions about the data. This could be used, for example, as a way to include information about the coupling between the multi channels measurements to the model. This would, however, necessitate proper modelling of the head as volume conductor.

We will use generic basis functions (Gaussian shaped humps with different time delays) and add data specific information to the estimation in the form of a priori information.

III. Subspace regularization

The task is to estimate the parameters θ in (1) based on the measurements z. Generalized least squares solution for the parameters θ is

$$\hat{\theta} = \arg\min_{\theta} \left\{ \|(z - H\theta)\|^2 + \alpha^2 \|L\theta\|^2 \right\}. \tag{2}$$

The equation is clearly a modification of the ordinary least squares solution $\hat{\theta}^{\mathrm{LS}} = \arg\min_{\theta}\{\|(z - H\theta)\|^2\}$ to the direction in which the semi norm $\|L\theta\|$, the so-called *side constraint*, gets smaller. It is easy to show that the regularized solution can be written in the form

$$\hat{\theta} = (H^T H + \alpha^2 L^T L)^{-1} H^T z \tag{3}$$

Now we can add the data specific information to the estimation by forming appropriate side constraint. We want our estimates to be close to the subspace $\mathcal{S}$ spanned by the first p (typically $p = 4$) eigenvectors of the data correlation matrix. Let these eigenvectors be the columns of the matrix $H_{\mathcal{S}}$. Now the projection of $s_i = H\theta_i$ onto $\mathcal{S}$ is $(H_{\mathcal{S}}H_{\mathcal{S}}^T)H\theta_i$. The distance of s_i from $\mathcal{S}$ can be written in the form $\|(I - H_{\mathcal{S}}H_{\mathcal{S}}^T)H\theta_i\|$. Remembering that we should construct such a matrix L that the side constraint $\|L\theta_i\|$ in (2) is small for all expectable θ_i, we thus select $L = (I - H_{\mathcal{S}}H_{\mathcal{S}}^T)H$. Since $L^TL = H^T(I - H_{\mathcal{S}}H_{\mathcal{S}}^T)H$, the desired solution for the parameters θ can be written in the form

$$\hat{\theta} = (H^TH + \alpha^2 H^T(I - H_{\mathcal{S}}H_{\mathcal{S}}^T)H)^{-1}H^Tz \qquad (4)$$

The estimates for the evoked potentials can be calculated with

$$\hat{s} = H\hat{\theta} \qquad (5)$$

This is called the subspace regularized solution [1].

In the case of multi channel measurements we can take the coupling between the channel into account by concatenating measurements from different channels together. The data correlation matrix will then also contain information about coupling between different channels. We will use diagonal block matrix as observation matrix H, in which each block is equal to single channel case. This way our model does not depend on the properties of the head or on the locations of the electrodes.

IV. Results

To study the effect of the regularization parameter α to the presented methods we calculated the estimates for simulated data with different values of the parameter α. We used single channel method and for multi channel case with five channels. Our data consist of 70 simulated responses. Evoked responses were modeled by calculating the potential field on the surface of the head caused by two dipole sources as their magnitude varied as function of time. Background EEG was taken from real measurements.

We calculated the RMS error between the estimates and noiseless simulations. Results for channels FZ, CZ, PZ and C3 as function of the regularization parameter α are shown in the top row of the Fig. 1 a). Multi channel method seems to have a little bit better overall performance.

The main benefit of the multi channel approach is its ability to preserve amplitude ratios between channels. This can be seen from the bottom row of the Fig. 1 a) where the RMS error of the amplitude difference of P3 peak between CZ and PZ and between CZ and C3 is shown as function of the regularization parameter. In Fig. 1 b) are shown the estimated P3 peak amplitude differences between CZ and PZ with two different values of alpha. It can be seen that the variation of the estimates is much smaller in the multi channel case.

V. Discussion

Methods for estimation of single trial ERPs both for single channel and multi channel data are described. The effect of the regularization parameter α to the estimates obtained with both of the methods is studied. Both methods are shown to be robust for small changes of the value of the regularization parameter.

The multi channel method seems to be better in preserving the amplitude differences between channels. This enables comparison of the single trial amplitudes between different channels.

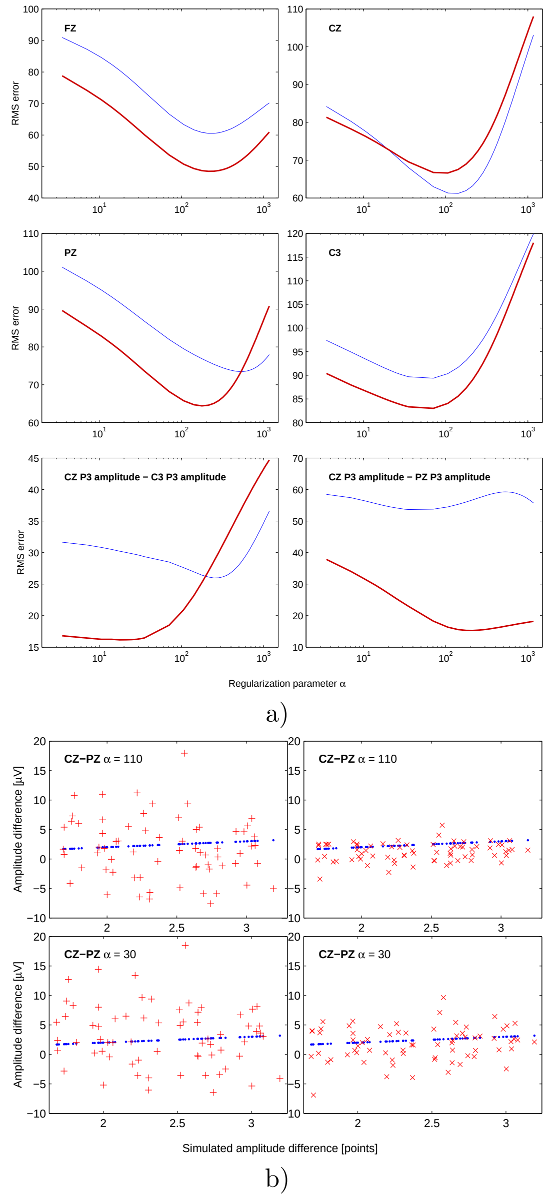

Fig. 1. a) RMS error of estimates of channels FZ, CZ, PZ and C3. RMS error of amplitude difference between channels CZ and PZ and channels CZ and PZ as function of α. Thin (blue) line corresponds to single channel method and thick (red) line to multi channel method. b) Amplitude difference of the estimates as function of simulated amplitude difference (dots). Single channel method estimates are in left column and multi channel method estimates are in right column.

References

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