Time-Varying Reconstruction in Single Photon Emission Computed Tomography

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ABSTRACT

We propose a new method for reconstruction of images in single photon emission computed tomography (SPECT), when the activity distribution of the object is time-varying. The activity evolution is modeled with the first-order Markov model, and linear observation model is used to characterize the measurement system. The state-space representation of the measurement sequence reduces to an ill-conditioned state estimation problem, which is solved recursively using the Kalman filter and smoother algorithms. The method is evaluated using simulations.

1. INTRODUCTION

In dynamic emission tomography studies the goal is to estimate the time-varying activities of certain organs. Traditionally this is done by reconstructing images corresponding to projections over 180° or 360° and assuming that the tracer distribution is stationary during the measurement of the used projection set. Images are then used to form the time-activity curves (TAC's) of the organs and kinetic parameters of suitable model can be estimated based on TAC's.

However, in practical situations and with reasonable patient doses, the total measurement time of one projection can be approximated to be as long as 30 s, or more. This time is remarkably long in, for example, myocardial imaging in which the effective half-lives of certain type of tracers can be some minutes. Therefore the time-variation of the object should be taken into account in the reconstruction process. Methods that take into account the time-variation include direct kinetic parameter estimation, for example [1] [2], and reconstruction of images corresponding to each measurement [3], [4].

We propose a new method to solve the time-varying emission tomography imaging problem. Recursive method estimates the tracer distribution corresponding to each individual projection measurement.

2. METHODS

In time-varying case with discrete space and time the SPECT measurements can described as

$$p_t = H_t\theta_t + v_t, \qquad (1)$$

where vector θ_t describes the activity distribution at time t, matrix H_t is the device-dependent projection operator that maps the activity distribution to the projection measurements p_t and v_t is additive gaussian noise term. Certain types of spatial prior information (known uniform areas, image pixels outside object, etc.) can be included in the observation matrix H_t. In strict sense the noise in SPECT measurements is neither additive or gaussian, however, many of the conventional reconstruction methods make this approximation and feasible results can be obtained without the use of the correct Poisson statistics for the measurement noise.

Typically a dynamic study consist of several rotations around the object with uniformly spaced projection measurements. For simplicity we assume that the time-discretization is equal to the projection measurements, that is, that the activity distribution is constant during the measurement of a single projection.

The time-evolution of the tracer distribution can be modeled in several different ways. Two of the most common types of models are the compartmental model and the diffusion model. In a 1-compartment model one may assume that the object consists of compartments (tissue types, blood pool etc.) with uniform activity. The uptake $\mu^m(t)$ of the activity in compartment m is described by the following equation

$$\mu^m(t) = k_u^m \int_0^t B(\tau)e^{k_w^m(t-\tau)}d\tau \qquad (2)$$

where k_u and k_w are the uptake and washout coefficients of the compartment m and $B(t)$ is the activity of the blood, which usually is assumed to be known. The total activity of a compartment can be presented as $\theta^m(t) = \mu^m(t) + f_v^m B(t)$, where f_v^m is the vascular fraction of the tissue type of compartment m.

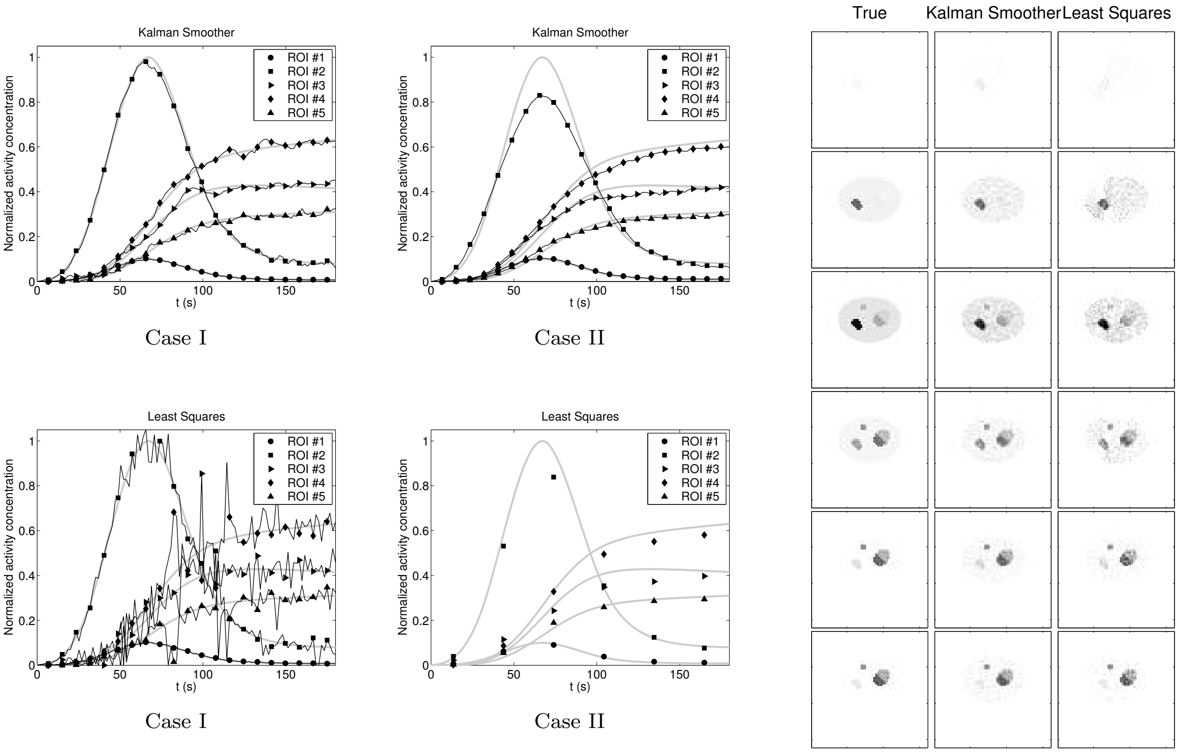


Fig. 1. Estimated TAC's of the compartments with known ROI's (Case I) and without ROI's (Case II). Simulated TAC's are shown with thick lines. Far right are given the estimated images in Case II corresponding to times, when LS-estimates are available. Time runs in vertical direction.

For the diffusion may written

$$\frac{\partial \theta(r,t)}{\partial t} = \nabla \cdot (\kappa(r) \nabla \theta(r,t)) + J(r,t) \tag{3}$$

where $\kappa(r)$ is the diffusion tensor and J known activity input function. The tracer evolution obeying either compartmental model Eq. (2) or diffusion model Eq. (3) can be described with a first order Markov model

$$\theta_{t+1} = F_t \theta_t + w_t + J_t, \tag{4}$$

where F_t is the state transition matrix that depends on the evolution model and w_t is the state noise term [5]. In the following we will neglect the input J_t.

Equations (1) and (4) together form the state-space model for dynamic emission tomography and the time-varying activity distribution θ_t can be solved using the Kalman smoother algorithm. First, the Kalman filter can be computed as

$$\hat{\theta}_{t|t-1} = F_{t-1} \hat{\theta}_{t-1|t-1} \tag{5}$$

$$C_{t|t-1} = F_{t-1} C_{t-1|t-1} F_{t-1}^T + C_{w_{t-1}} \tag{6}$$

$$K_t = C_{t|t-1} H_t^T \big(H_t C_{t|t-1} H_t^T + C_{v_t} \big)^{-1} \tag{7}$$

$$\hat{\theta}_{t|t} = \hat{\theta}_{t|t-1} + K_t \big(p_t - H_t \hat{\theta}_t \big) \tag{8}$$

$$C_{t|t} = C_{t|t-1} - K_t H_t C_{t|t-1} \tag{9}$$

and using the estimates $\hat{\theta}_{t|t}$, predictors $\hat{\theta}_{t|t-1}$ and covariance matrices, the Kalman smoother estimates are obtained by running the following iteration backward in time

$$A_{t-1} = C_{t-1|t-1} F_{t-1}^T C_{t|t-1}^{-1} \tag{10}$$

$$\hat{\theta}_{t-1|T} = \hat{\theta}_{t-1|t-1} + A_{t-1} \big(\hat{\theta}_{t|T} - \hat{\theta}_{t|t-1} \big) \tag{11}$$

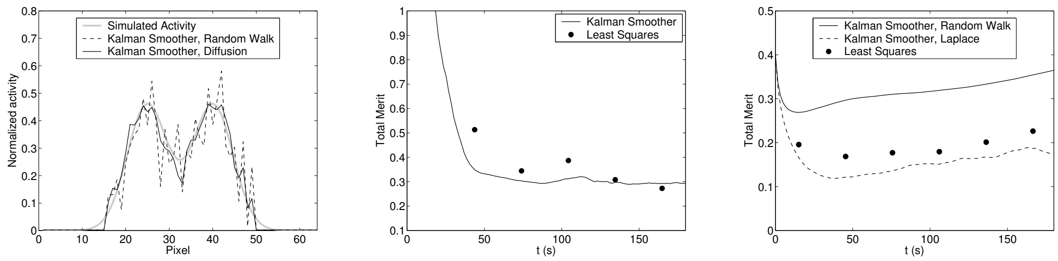
$$C_{t-1|T} = C_{t-1|t-1} + A_{t-1} \big(C_{t|T} - C_{t|t-1} \big) A_{t-1}^T \tag{12}$$

The activity distribution θ_t is non-negative by definition but this constraint can not be implemented in the Kalman filter directly. Non-negative estimates are obtained by finding the non-negative maximizer of the gaussian probability density $\mathcal{N}(\hat{\theta}_{t|T}, C_{t|T})$ for each t, i.e. solving the minimization problem $\big(W_t^T W_t = C_{t|T}^{-1}\big)$

$$\hat{\theta}_{t|T}^{*} = \arg\max_{\theta \geq 0} \Big\{ \| W_t (\theta - \hat{\theta}_{t|T}) \|^2 \Big\}. \tag{13}$$

3. RESULTS

Both compartmental and diffuse transportations are simulated and the proposed method is compared to sequential application on non-negativity constrained LS-estimates. Projections are measured with 10° increments over three full rotations around the object. The conventional LS-estimates can be obtained using all of the data from 180° rotation for single image. The error of estimates is described by the merit [3] $M_t = \sqrt{\sum_j \big(\theta_{j,t} - \hat{\theta}_{j,t}\big) / \sum_j \theta_{j,t}^2}$ where j denotes the pixels in reconstructed field of view. Further details of the simulations are given in [5].



I II III

Fig. 2. I) Sample profiles of the Kalman smoother estimates in case of diffuse transport. II) Merits of estimates in Case II of compartmental transport. III) Merits of estimates for diffuse transport.

3.1 Compartmental transport

For compartmental transport 5-compartment phantom is used. One of the compartments (ROI #2) represents the blood pool that is associated with input function $B(t)$, the uptakes of the other compartments are computed using Eq. (2). No prior information about the time-variation is used, thus a random-walk model $F_t = I$ is used in the reconstructions. Also $B(t)$ is unknown in reconstructions, altough this sort of prior information could be easily included in the reconstruction process.

If the locations of the compartments are known (based on CT or MRI) the mean activity of the compartments can be estimated directly by modifying the observation matrix and the number of unknowns is reduced remarkably. In this case (Case I, Fig. 1) the model is usually overdetermined and also the LS-estimates are available for every observation. In Case II Fig. 1, when the information about the compartments is absent, the images are reconstructed pixel-by-pixel and the mean activities are computed after the reconstruction.

3.2 Diffuse transport

Homogeneous diffusion of a ring-shaped tracer distribution is simulated using Eq. (3) and the estimates of the activity are computed using both random walk model and diffusion model with underestimated diffusion coefficient (labeled "Laplace" in figures) as well as with constrained LS-method. Fig. 2 I and Fig. 3 show that the use of correct evolution model reduces the effect of noise in the estimates and makes the estimates smoother than those obtained with non-negative LS or random-walk Kalman smoother.

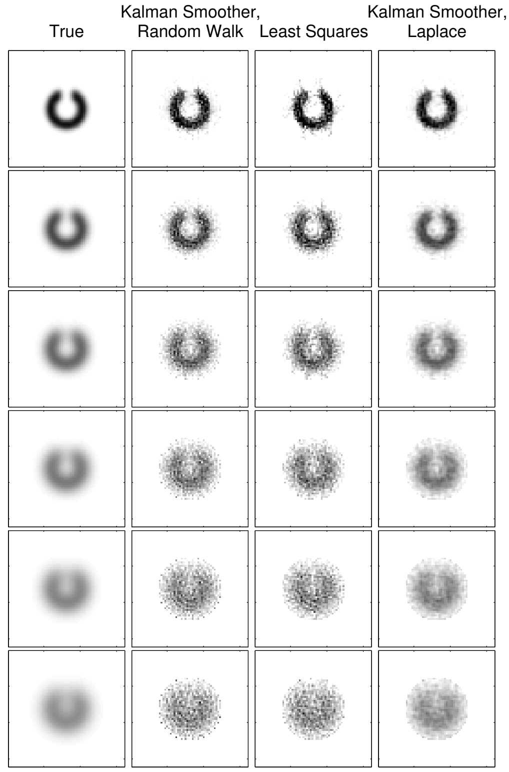


Fig. 3. Diffuse transport. Estimated images at times, when LS-estimates are available. Time runs in vertical direction.

4. DISCUSSION

We have proposed a new recursive estimation method for time-varying emission tomography imaging problem. Method is capable of reconstructing images corresponding to each projection measurement and performs at least as well as sequential application of LS-estimates using 180° measurements (Fig. 2). Additional spatial (compartments) and temporal (evolution models) prior information can be taken into account in the reconstruction easily.

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