Time-Varying Reconstruction in Single Photon Emission Computed Tomography

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Outline of the presentation

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- Time-varying case
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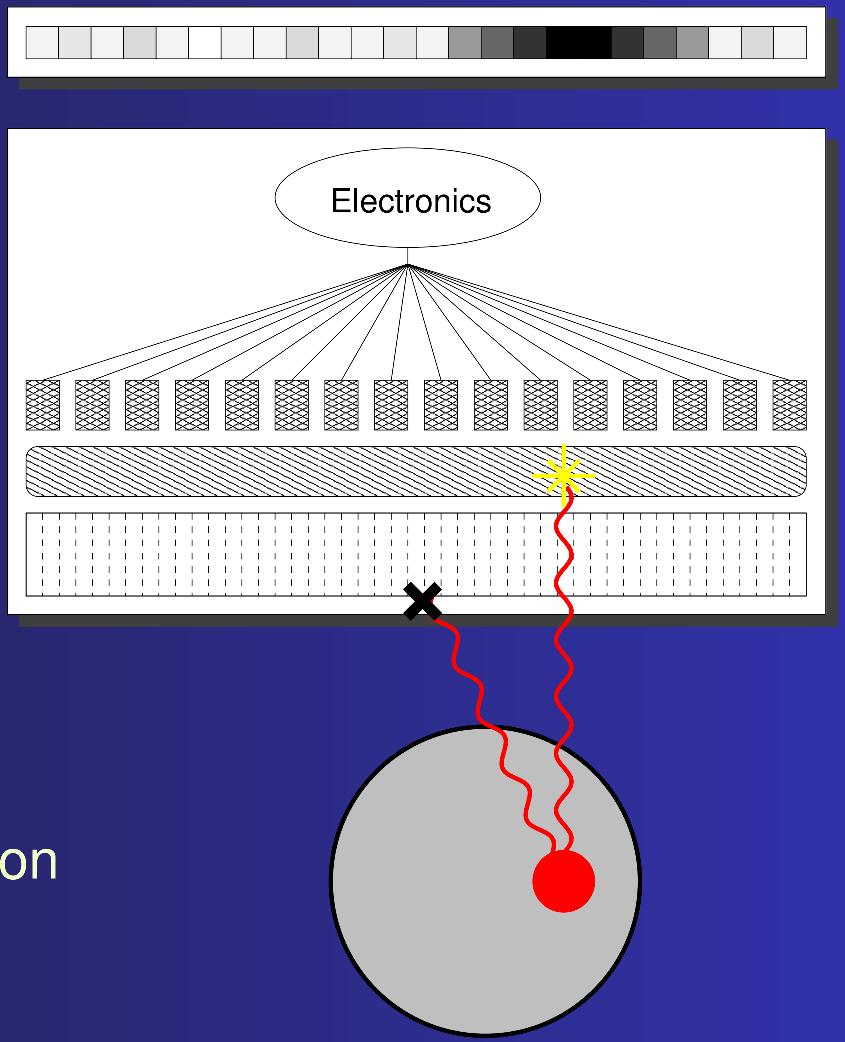
Introduction

- In tomographic image reconstruction one basic assumption is the time–independence of the object to be imaged. In some studies the change of the object is, however, the object of interest.
- In single photon emission tomography the requirement of the time–independence is often not fulfilled. Several methods have been developed to speed up the acquisition process and thereby to reduce the effect of time–variation of the object.
- Current dynamic SPECT studies are based on the set of stationary images that are used to determine functional parameters of metabolism.
- Truly time–varying reconstruction would increase the time–resolution of the studies and reduce the artifacts caused by the assumption of time–independence.

Principle of SPECT measurements

Gamma camera in 2D

- Aim is to measure a parallel projection of the photons emitted from the object
- (Ideal) **collimator** blocks photons that are not parallel to its holes
- Photons are captured by the **scintillation crystal**
- "Signal" is amplified by the **photomultiplier tubes**
- In 2D result is a row vector of photon counts



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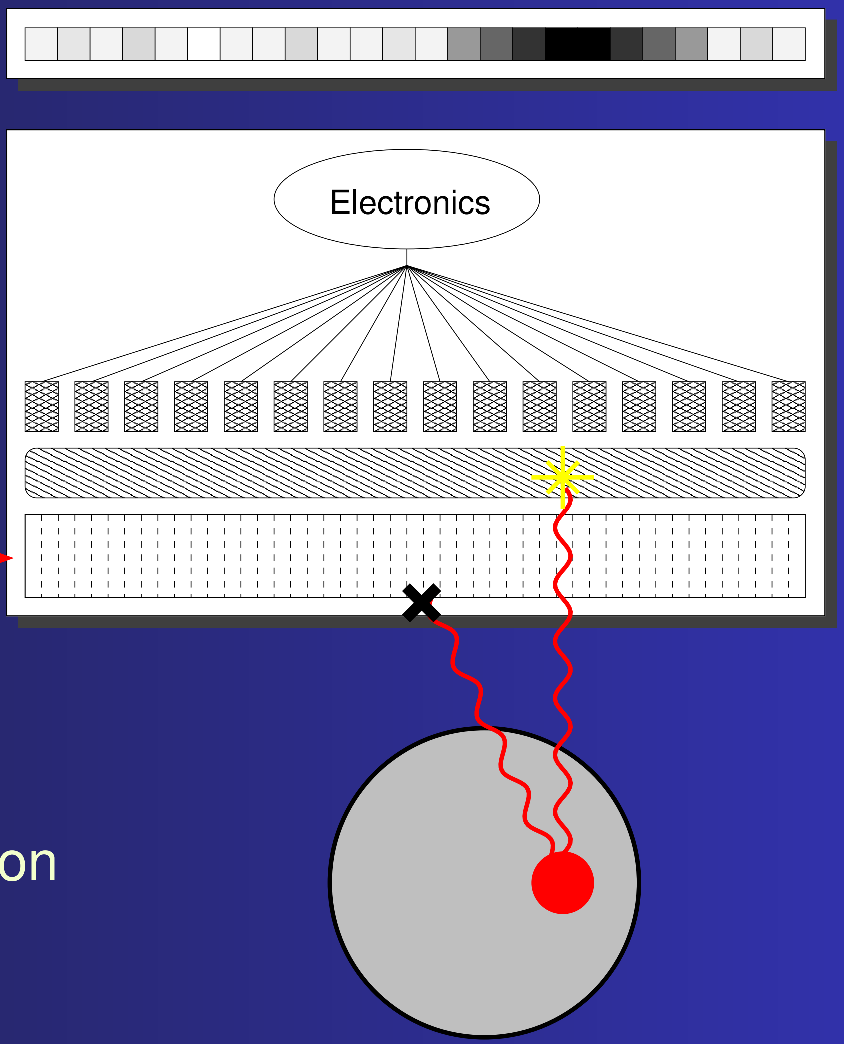

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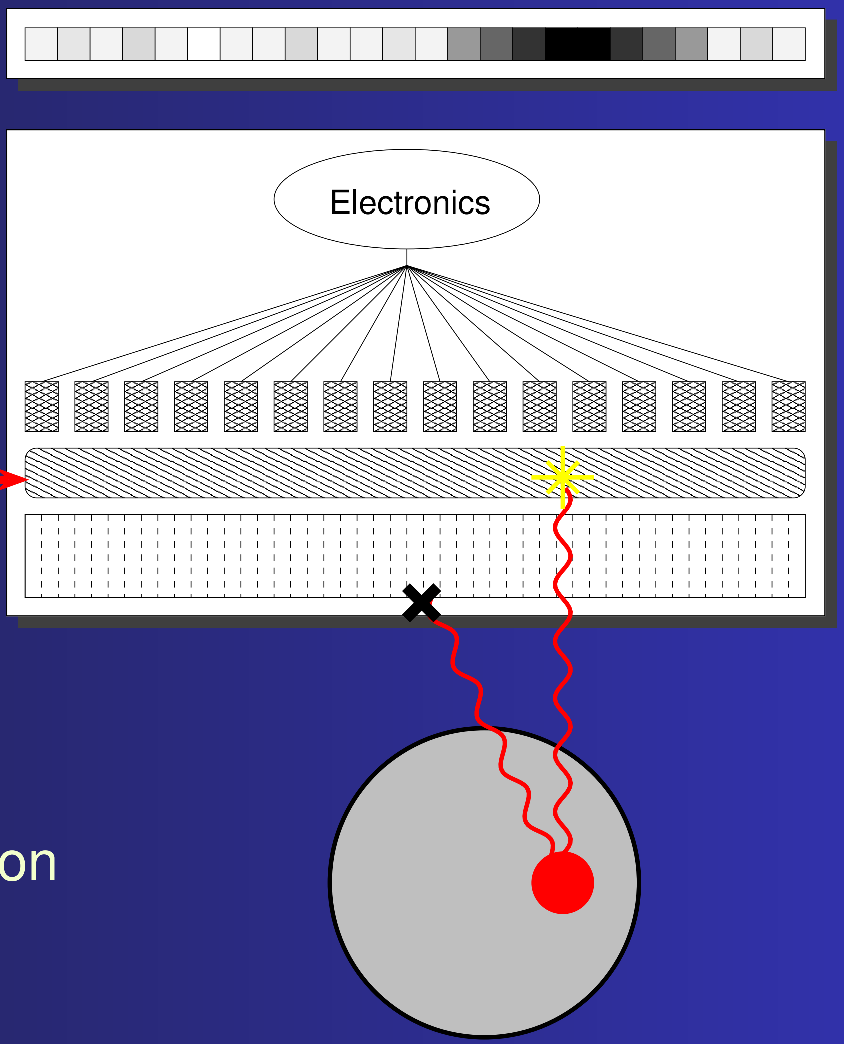

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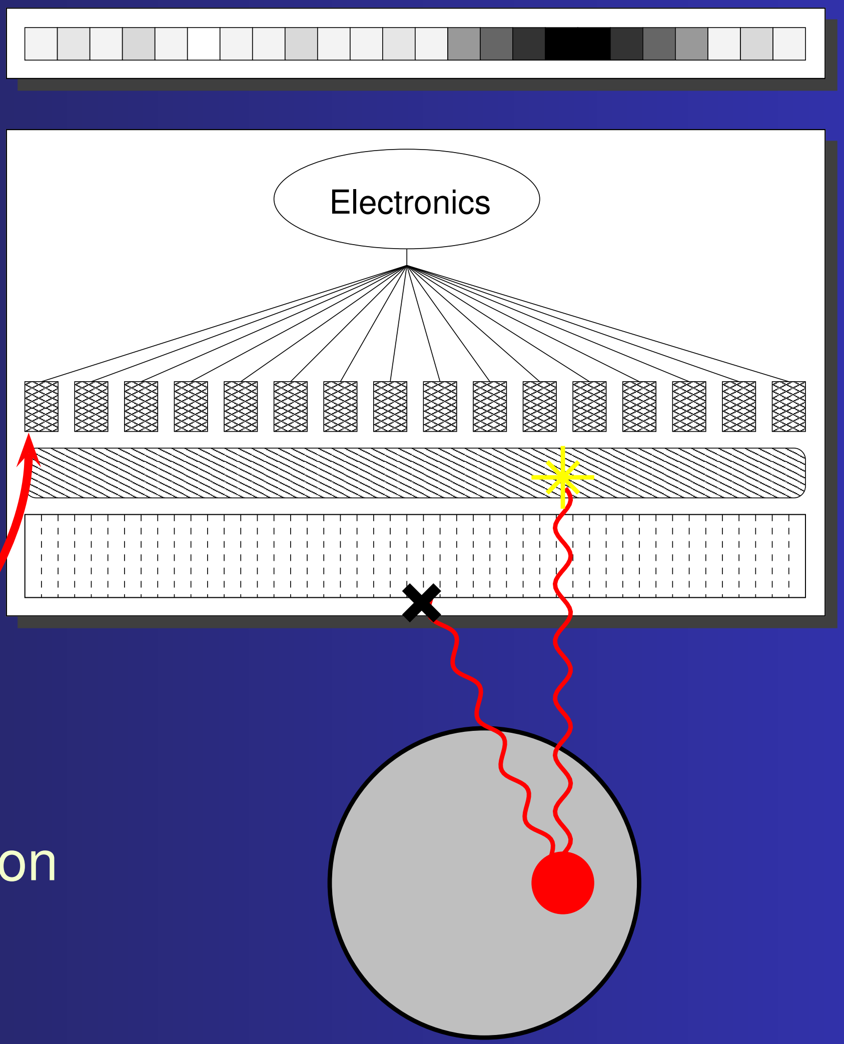
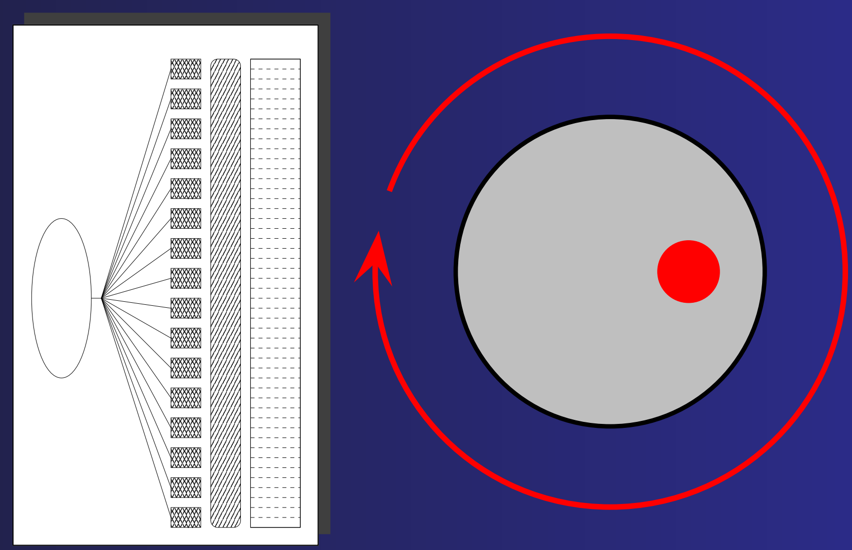


Tomographic measurement

- Projections are measured around the object by rotating the gamma camera for example 10° between the measurements.



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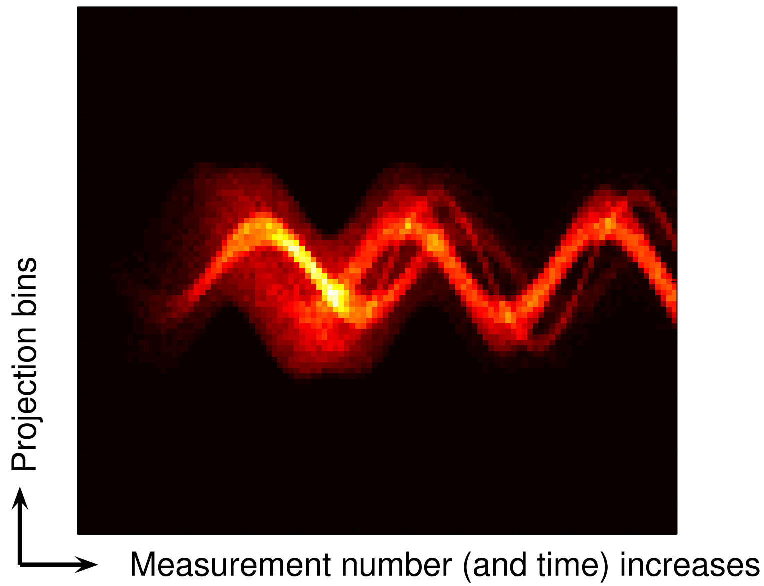
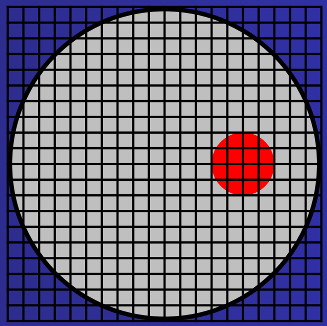
- In 2D case each measurement is a vector. Single measurements are often combined and presented as a sinogram.



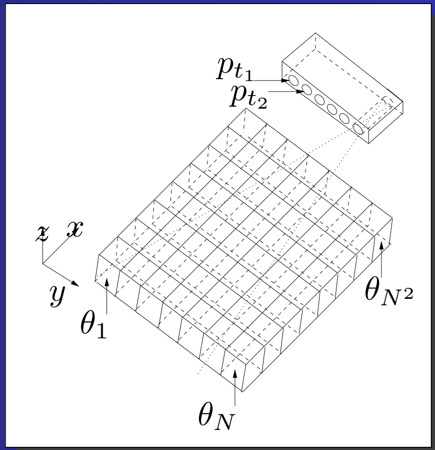
Tomographic measurement

- Projections are measured around the object by rotating the gamma camera for example 10° between the measurements.
- In 2D case each measurement is a vector. Single measurements are often combined and presented as a sinogram.
- In stationary case all of the projections image identical activity distribution and differ only in the perspective of the view.
- In this case the activity distribution can be reconstructed using classical projection methods, such as Filtered Back Projection, or using more sophisticated methods combined with the proper modeling of the measurement system.

Notations



- Usually the 2D image plane is discretized with N^2 square pixels and measurement operator, denoted as H, that is the mapping from discretized activity distribution θ to the projection p, is constructed.
- Construction of observation matrix H may include, for example the effects of geometry, attenuation, scatter and septal penetration in the collimator.
- Mapping is different for each measurement direction. In the following we will denote the observation matrix at time t with H_t and the corresponding observed projection vector with p_t. Thus t denotes both time and direction of the camera head.



Stationary case

- By combining all observations, linear observation model for stationary SPECT can be written in the form

$$\begin{pmatrix} p_1 \\ \vdots \\ p_T \end{pmatrix} = \begin{pmatrix} H_1 \\ \vdots \\ H_T \end{pmatrix} \theta,$$

  where p_t is the measured projection at time t, θ is the vectorized activity distribution (image) and H_t is the projection operator that maps the activity distribution to the measurements at time t.

- Even in stationary case the observation model of SPECT is highly underdetermined and the reconstruction of the activity distribution requires novel mathematical methods.

- In SPECT the error is neither additive nor Gaussian. However, if these assumption are made, the observation equation is of the form

$$p = H\theta + v,$$

where v is the measurement error and H the combined projection operator and p denotes the stacked observations. Estimator for θ can then be obtained, for example, using constrained least squares approach:

$$\hat{\theta}_{\mathrm{LS}} = \arg\min_{\theta > 0} \left\{ \|p - H\theta\|^2 \right\}$$

with additional constraints like Tikhonov regularization.

- Another approach is to take account the Poisson-statistics of the observations using methods like Maximum Likelihood Expectation Maximization (MLEM) algorithm.

Time-varying case

- In many cases the activity distribution is not stationary during the full tomographic measurement (180 or $360°$ rotation)
- If standard reconstruction methods are used to estimate the rapidly changing object, errors are generated due to the inconsistency of the projection data.
- Thus, the change of the activity distribution should be taken into account in the reconstruction process.
- Instead of single distribution we must reconstruct multiple images using the same amount of data that is used with the standard methods.

State-space equations

- In time-varying case we approximate that the activity distribution is constant during the measurement of a single projection.

$$p_t = H_t\theta_t + v_t$$

- Observations cannot be combined as in stationary case.
- In order to reconstruct the time-varying activity distribution $\theta_{1,\ldots,T}$, a model of the time-variation is required.
- The simplest modeled for the time-variation is the first order Markov model

$$\theta_{t+1} = F_t\theta_t + w_t$$

where F_t is the state transition matrix and w_t the state noise.

Transportation models

- Both compartmental transport and diffuse transport can be represented with the first order Markov model.
- 1-compartment model can be used to model the activity uptake μ of tissues when blood activity $B(t)$ is known:

$$\mu^m(t) = k_u^m \int_0^t B(\tau) e^{k_w^m (t-\tau)} d\tau$$

where k_u^m and k_w^m are the uptake and washout coefficients of the tissue type m.

- The activity in tissue m is obtained by taking into account the vascular fraction f_v^m of the corresponding tissue type

$$\theta^m(t) = \mu^m(t) + f_v^m B(t)$$

- In case of diffuse transport the time–evolution can be written as a non–homogeneous partial differential equation

$$\frac{d\theta(\mathbf{r},t)}{dt} = \nabla \cdot (\rho(\mathbf{r})\nabla\theta(\mathbf{r},t)) + J(\mathbf{r},t)$$

  In case of isotropic homogeneous diffusion $\rho(\mathbf{r})$ is a scalar constant and $\nabla \cdot (\rho(\mathbf{r})\nabla\theta(\mathbf{r},t)) = \rho\nabla^2\theta(\mathbf{r},t)$. The operator $\rho\nabla^2$ equals to linear spatial filtering of the activity distribution.

- By approximating the derivatives with the first difference, the state equations for both compartmental and diffusion models will be of form

$$\theta_{t+1} = F\theta_t + GJ_t$$

  where the state transition matrix F and the known input GJ_t are constructed according to the desired model.

Solution of the state-estimation problem

- The time-varying distribution can be solved by applying the Kalman Filter to the state–space equations
- The problem is to find the minimum mean square estimator $\hat{\theta}_t$ for state θ_t given the observations up to time t, $p_1, p_2, \ldots, p_t$.
- Kalman Filter equations can be written in the form

$$
\begin{aligned}
\hat{\theta}_{t|t-1} &= F_{t-1}\hat{\theta}_{t-1|t-1} \\
C_{\tilde{\theta}_{t|t-1}} &= F_{t-1}C_{\tilde{\theta}_{t-1|t-1}}F_{t-1}^T + C_{w_{t-1}} \\
K_t &= C_{\tilde{\theta}_{t|t-1}}H_t^T(H_tC_{\tilde{\theta}_{t|t-1}}H_t^T + C_{v_t})^{-1} \\
C_{\tilde{\theta}_t} &= \left(I - K_tH_t^T\right)C_{\tilde{\theta}_{t|t-1}} \\
\hat{\theta}_{t|t} &= \hat{\theta}_{t|t-1} + K_t\left(p_t - H_t^T\hat{\theta}_{t|t-1}\right)
\end{aligned}
$$

- In SPECT the reconstruction is done usually off–line and all the observation $p_1, p_2, \ldots, p_T$ are available. The algorithm that computes the estimate based on all observations, i.e. $\hat{\theta}_{t|T}$ is called fixed interval Kalman Smoother. In addition to the Kalman Filter equations so called backward run gain matrices

$$A_{t-1} = C_{\tilde{\theta}_{t-1}} F_{t-1}^T C_{\tilde{\theta}_{t|t-1}}^{-1}$$

must be stored during filtering and the Smoother estimate can be computed applying equation

$$\hat{\theta}_{t-1|T} = \hat{\theta}_{t-1|t-1} + A_{t-1}(\hat{\theta}_{t|T} - \hat{\theta}_{t|t-1})$$

in time–reversed (backward) order.

Additional constraints

- The activity distribution is non-negative by definition. This constraint cannot be included in the Kalman filter directly.
- To find the optimal non-negative estimates of the activity distribution, we use the Kalman filter estimates and the error covariances, provided by the filter, to find an estimate

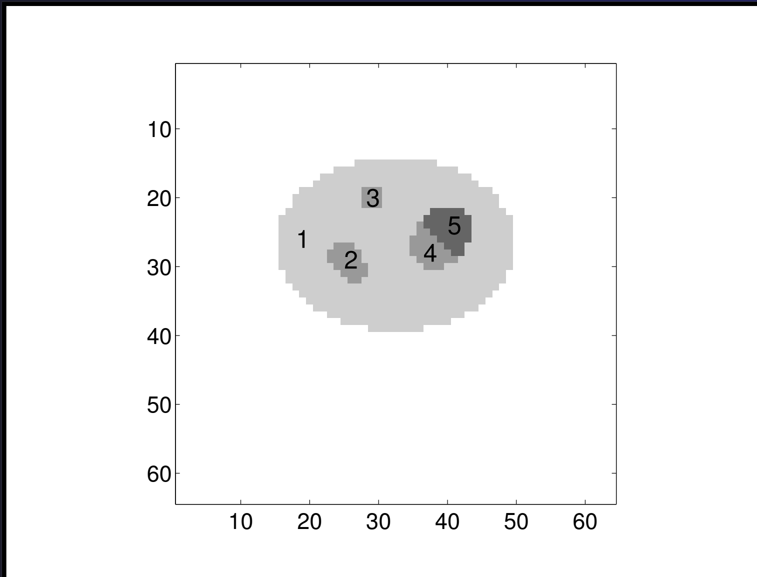
$$\hat{\theta}^*_{t|t} = \arg\max_{\theta > 0} \left\{ \exp(-\frac{1}{2}(\theta - \hat{\theta}_{t|t}) C^{-1}_{\tilde{\theta}_{t|t}} (\theta - \hat{\theta}_{t|t})) \right\}$$

that is, we select the most probable non-negative estimates at each time-step from the distribution $\mathcal{N}(\hat{\theta}_{t|t}, C_{\tilde{\theta}_{t|t}})$ defined by the Kalman filter.

Results

- Proposed method is evaluated using simulations
- Projections from a time-varying activity distribution are acquired using 2D parallel hole collimator and single head camera. Attenuation and scatter are not modeled.
- Our simulated measurements include 3 full rotations using 10° steps. Observation data is Poisson distributed, also additive Gaussian noise is added to the simulated observations.
- Conventional method (positivity constrained LS) can be used to reconstruct single image using 180° data, that is, using 18 projections.
- Kalman smoother is used to estimate the activity distribution corresponding to each projection measurement.

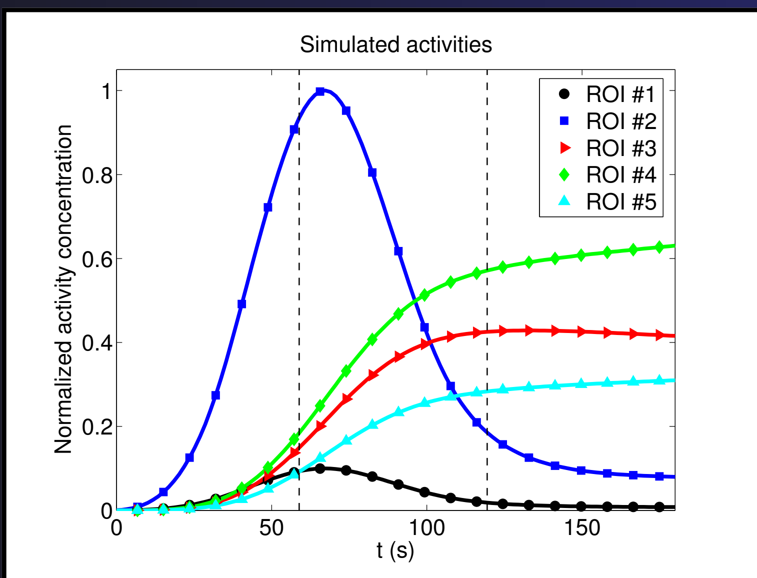
> Static imaging sequence might include full 360° rotation with 3° steps giving 120 projections to be used for reconstruction of a single image

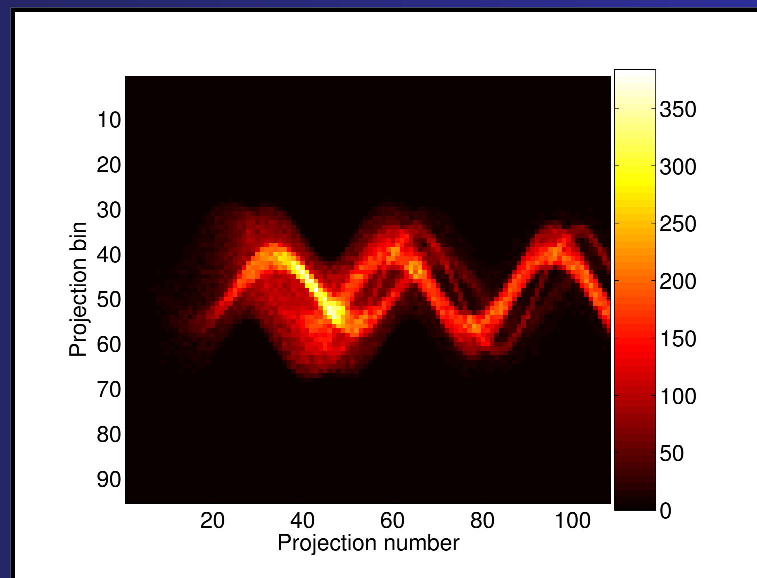


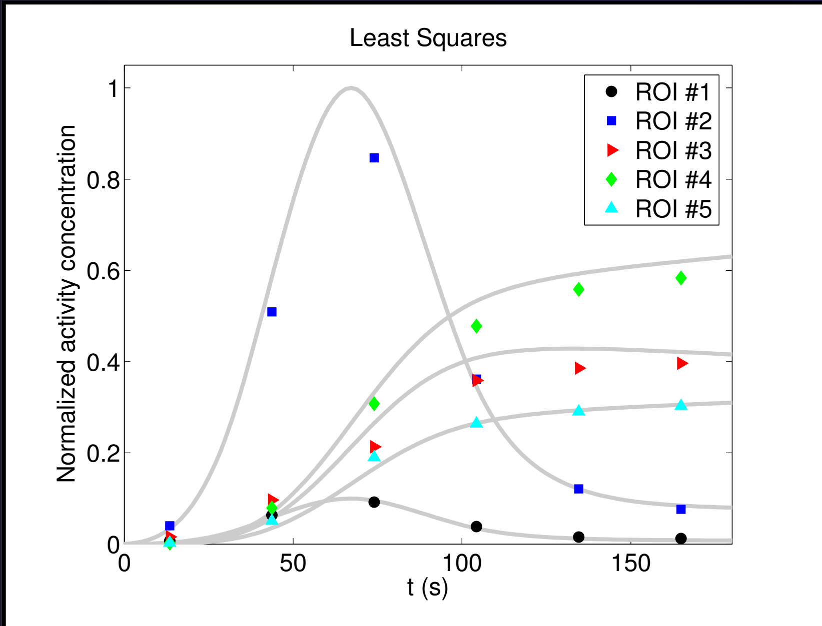
Phantom consists of 5 tissue types with spatially constant activity in each.

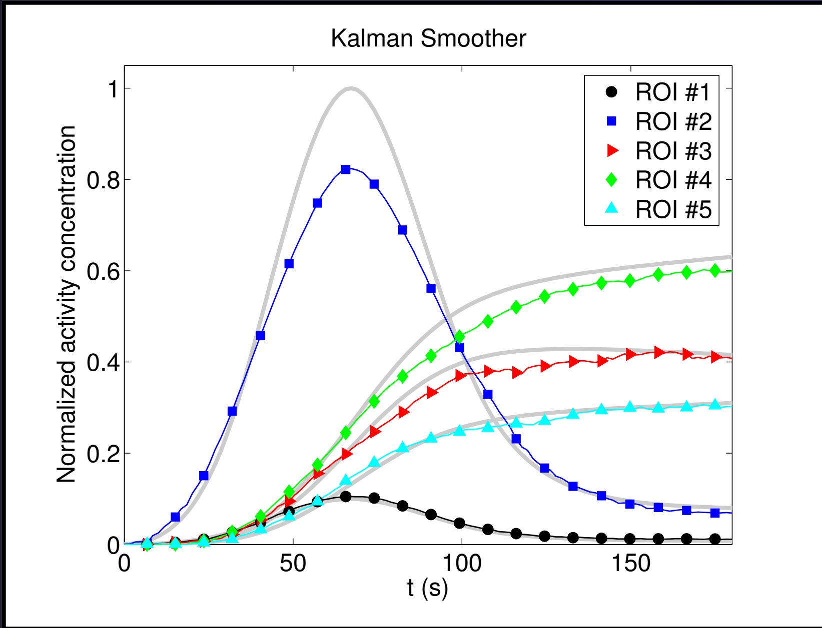
Activity/pixel within each tissue type is computed using given blood input function (time-activity curve of ROI #2) and the 1-compartment model

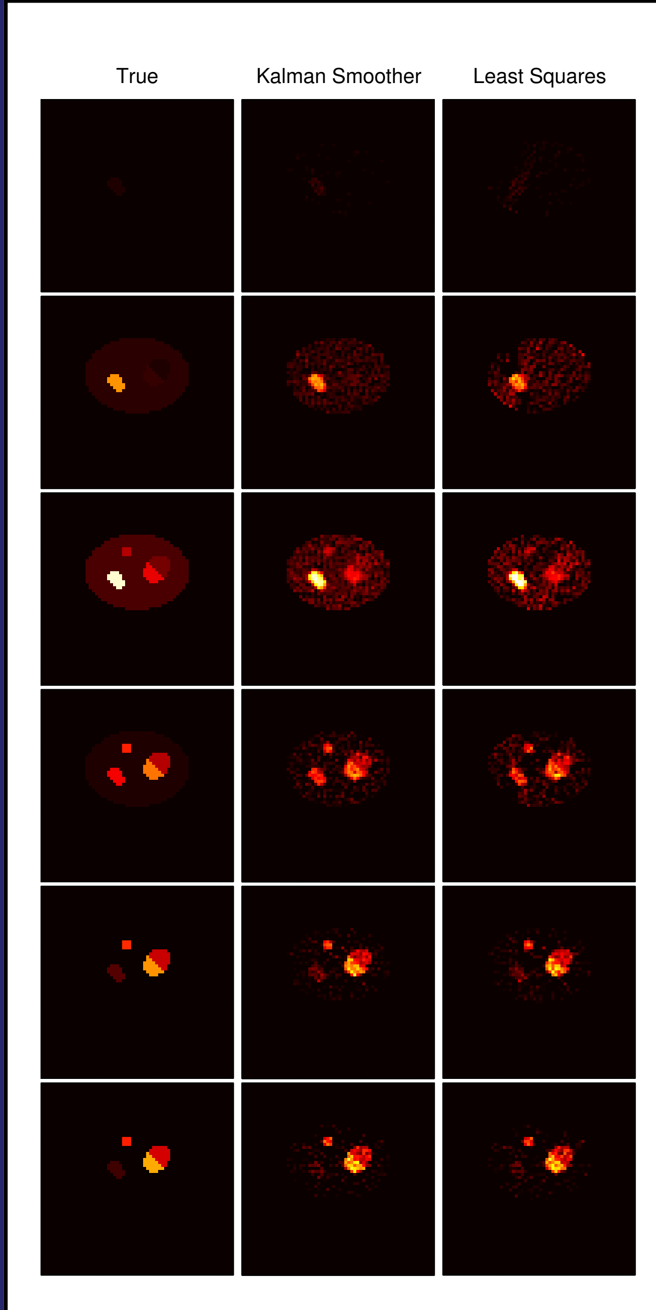
For the Kalman smoother the state equation is a random walk model, that is $F_t \equiv I$. This means that no information about the blood input or transportation is used in the reconstruction.






Least Squares
Normalized activity concentration
t (s)
ROI #1
ROI #2
ROI #3
ROI #4
ROI #5
0
0.2
0.4
0.6
0.8
1
0
50
100
150


Kalman Smoother
Normalized activity concentration
t (s)
ROI #1
ROI #2
ROI #3
ROI #4
ROI #5
0
0.2
0.4
0.6
0.8
1
0
50
100
150


True
Kalman Smoother
Least Squares

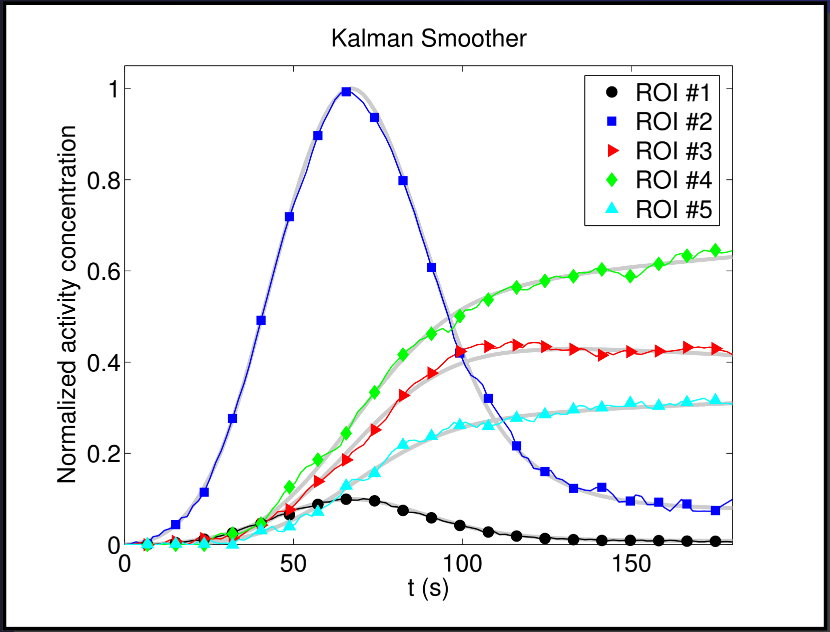
Reduction of parameters

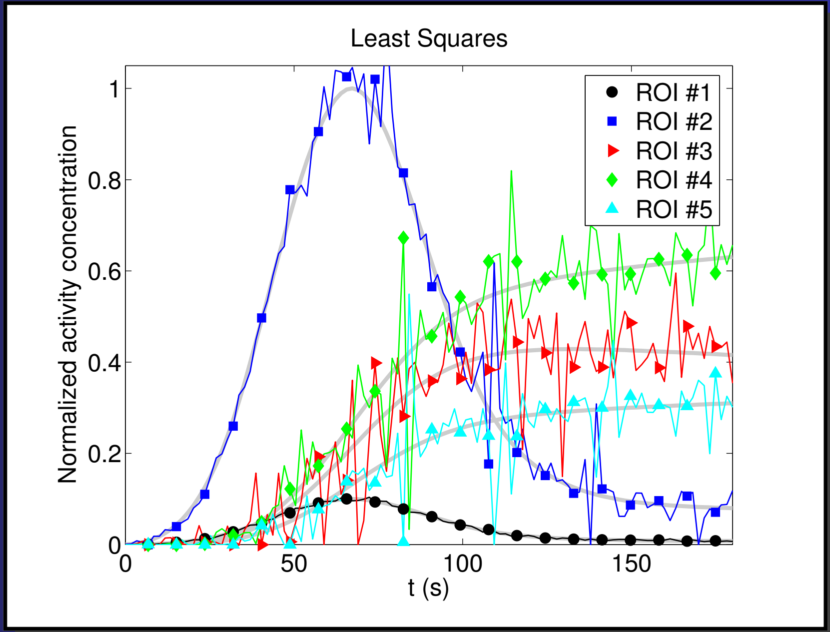
- In some cases we are interested in the total activity of some Regions of Interest (usually certain organs) rather than the activities of each pixel separately. By constructing a matrix R whose i'th column is the index set of the pixels that belong to i'th ROI, we can compute the total activity of each ROI by equation

$$\phi = R^T \theta$$

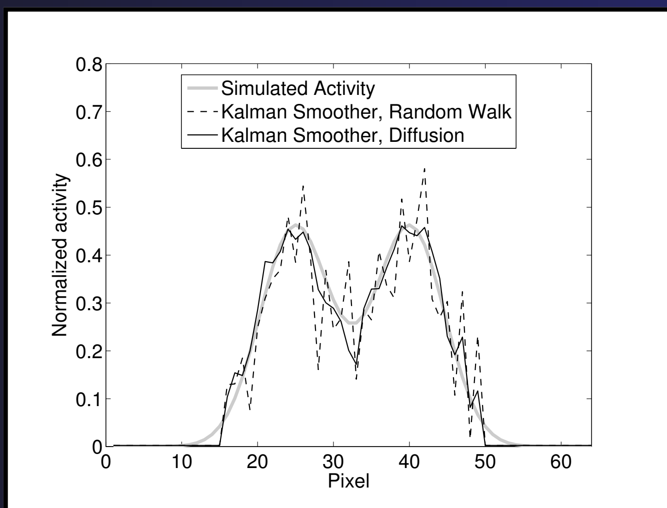
- If we assume that the activity distribution is approximately constant within each ROI, we can estimate the mean activity of the ROI's directly by modifying the observation model.
- In this case the number of unknown parameters is reduced to the number of ROI's (compare to the N^2 unknowns in pixel-based case.)

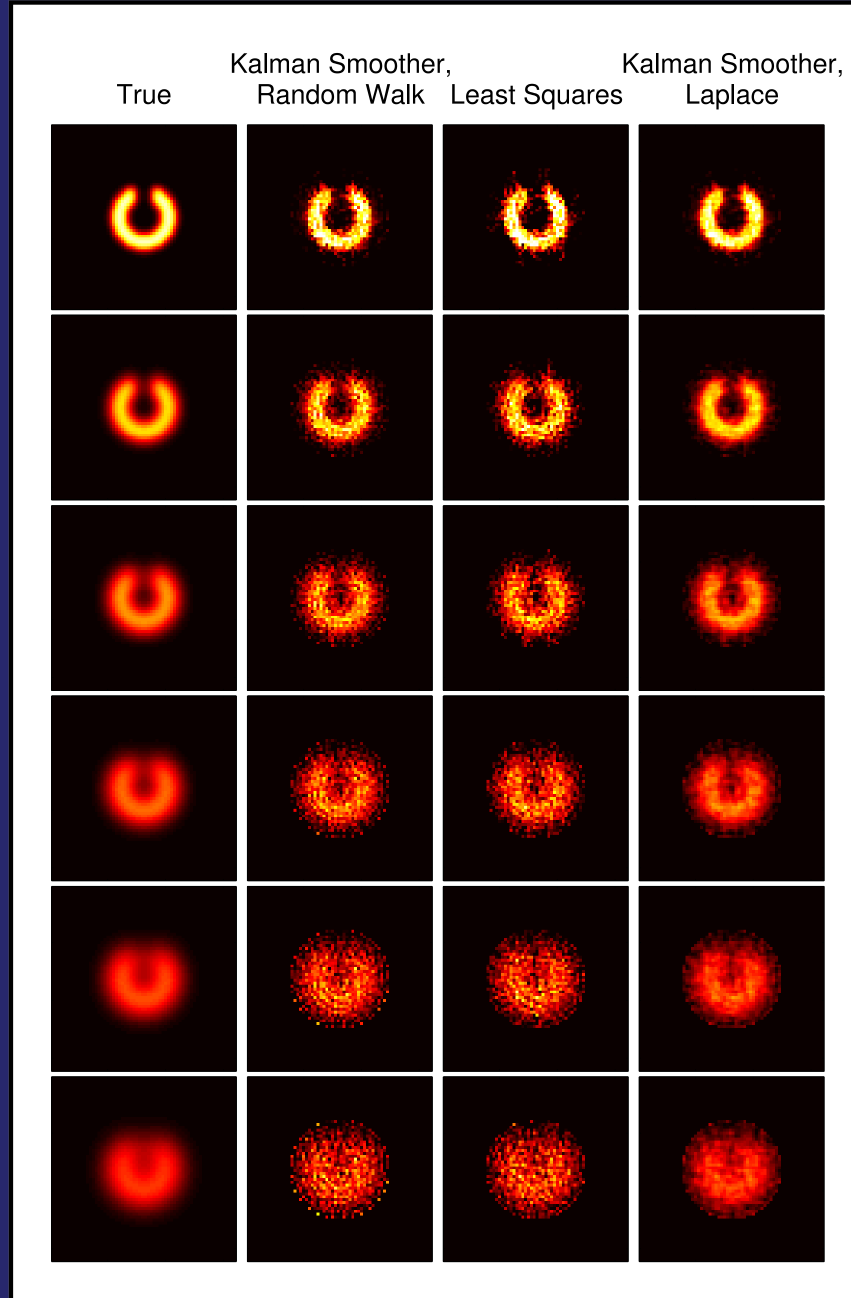
- When prior information about the ROI's is used, the model is overdetermined
- In this case also the LS-estimates can be computed sequentially corresponding to each measurement





- In case of diffuse transport we have used both random walk and diffusion models as the evolution model in the state space equations
- Use of diffusion model in the estimation clearly removes the effect of noise, compared to sequential LS and random walk estimates.





Conclusion

- A new method for reconstruction of dynamic SPECT images is presented
- Method is flexible, both spatial and temporal prior information can be easily formulated with the state-space equations.
- State estimation problem was solved with Kalman Smoother, which is (today) computationally too expensive when practical resolutions are considered.
- Particle filters could be used to take into account the Poisson statistics of the observation noise and include the non-negativity constraint directly in the filtering.

  $\rightarrow$ Computational cost would be still increased