Time-Varying ARMA modelling of Nonstationary EEG using Kalman Smoother Algorithm

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ABSTRACT

An adaptive autoregressive moving average (ARMA) modelling of nonstationary EEG by means of Kalman smoother is presented. The main advantage of the Kalman smoother approach compared to other adaptive algorithms such as LMS or RLS is that the tracking lag can be avoided. This advantage is clearly presented with simulations. Kalman smoother is also applied to tracking of alpha band characteristics of real EEG during an eyes open/closed test. The observed tracking ability of Kalman smoother, compared to other methods considered, seemed to be better.

1 INTRODUCTION

In the analysis of nonstationary EEG the interest is often to estimate the time-varying spectral properties of the signal. A traditional approach to this is the spectrogram method, which is based on Fourier transformation. Disadvantages of this method are the implicit assumption of stationarity within each segment and the rather poor time/frequency resolution. A better approach is to use parametric spectral analysis methods based on e.g. time-varying autoregressive moving average (ARMA) modelling. The time-varying parameter estimation problem can be solved with adaptive algorithms such as least mean square (LMS) or recursive least squares (RLS). These algorithms can be derived from the Kalman filter equations [1], [2].

In this paper we use the Kalman smoother algorithm in tracking of nonstationary properties of EEG. Kalman smoother is compared to LMS and RLS algorithms in tracking of alpha band characteristics of EEG measured during an eyes open/closed test. The Kalman smoother approach is also applied to the detection of alpha waves of EEG. The main advantage of the Kalman smoother algorithm compared to other adaptive algorithms is the fact that the tracking lag can be avoided. This is demonstrated with simulations. Kalman filter has been previously used in EEG analysis in e.g. [3], [4], [5].

2 METHODS

If the signal to be modelled is nonstationary it cannot be modelled as an output of a time-invariant system. It is natural in this case to assume that the system has time-varying parameters.

2.1 Time-varying linear regression

Here we use the time-varying autoregressive moving average ARMA(p,q) model for the signal

$$z(t) = -\sum_{j=1}^{p} a_j(t) z(t-j) + \sum_{k=1}^{q} b_k(t) e(t-k) + e(t) \quad (1)$$

where $a_j(t)$ and $b_k(t)$ are the time-varying ARMA parameters and $e(t)$ is the driving white noise process. By denoting

$$\theta_t = (-a_1(t), \ldots, -a_p(t), b_1(t), \ldots, b_q(t))^T \quad (2)$$

$$\varphi_t = (z(t-1), \ldots, z(t-p), e(t-1), \ldots, e(t-q))^T \quad (3)$$

the model can be written in the form

$$z_t = \varphi_t^T \theta_t + e_t \quad (4)$$

where $z_t = z(t)$ and $e_t = e(t)$. This is clearly a linear observation model, with φ_t^T being the observation matrix and e_t being the observation error. A typical description for the parameter variation when no *a priori* information is available, is the random walk model [6]. Thus for the parameters θ_t we write a state equation of the form

$$\theta_{t+1} = \theta_t + w_t \quad (5)$$

where w_t is a noise process. Equations (4) and (5) form a specific form of the general state space equations, with the input process w_t. Now the problem is to estimate the time-varying parameters θ_t, according to the state space model.

2.2 Kalman filter

The Kalman filtering problem is to find the minimum mean square estimator $\hat{\theta}_t$ for state θ_t given the observations $z_1, \ldots, z_t$. This has been shown to be equal to the conditional expectation value

$$\hat{\theta}_t = E\{\theta_t | z_1, \dots, z_t\} \quad (6)$$

We assume here the state and measurement noises w_t and e_t to be uncorrelated, zero mean, random processes with covariance matrices $C_{w_t} = \sigma_w^2 I$ and $C_{e_t} = \sigma_e^2 I$, so that the individual parameter evolutions are assumed to be independent. The initial state θ_0 is assumed to be uncorrelated with e_t and w_t with finite variance. The Kalman filter equations can be written in the form

$$\hat{\theta}_{t|t-1} = \hat{\theta}_{t-1} \quad (7)$$

$$C_{\tilde{\theta}_{t|t-1}} = C_{\tilde{\theta}_{t-1}} + C_{w_{t-1}} \quad (8)$$

$$K_t = C_{\tilde{\theta}_{t|t-1}} \varphi_t \left(\varphi_t^T C_{\tilde{\theta}_{t|t-1}} \varphi_t + C_{e_t} \right)^{-1} \quad (9)$$

$$C_{\tilde{\theta}_t} = \left(I - K_t \varphi_t^T \right) C_{\tilde{\theta}_{t|t-1}} \quad (10)$$

$$\epsilon_t = z_t - \varphi_t^T \hat{\theta}_{t|t-1} \quad (11)$$

$$\hat{\theta}_t = \hat{\theta}_{t|t-1} + K_t \epsilon_t \quad (12)$$

where $\hat{\theta}_{t|t-1}$ is the mean square estimator for state θ_t given the observations $z_1, \dots, z_{t-1}$, $\tilde{\theta}_t$ is the state estimation error $\tilde{\theta}_t = \theta_t - \hat{\theta}_t$ and K_t is the Kalman gain matrix. The adaptation of the filter is primarily affected by C_{w_t}.

2.3 Fixed-interval smoother

The fixed-interval smoothing problem is to determine estimates

$$\hat{\theta}_{t|T} = E\{\theta_t | z_1, \dots, z_T\} \quad (13)$$

for fixed T and for all t in the interval $1 \le t \le T$. The solution for this can be written in the form [7]

$$\hat{\theta}_{t-1|T} = \hat{\theta}_{t-1} + A_{t-1} \left(\hat{\theta}_{t|T} - \hat{\theta}_{t|t-1} \right) \quad (14)$$

$$A_{t-1} = C_{\tilde{\theta}_{t-1}} C_{\tilde{\theta}_{t|t-1}}^{-1} \quad (15)$$

where A_{t-1} includes the error covariances stored in the forward run of Kalman filter. Also the state estimates $\hat{\theta}_t$ and $\hat{\theta}_{t|t-1}$ need to be stored. The smoothed estimates $\hat{\theta}_{t-1|T}$ are then obtained by running the stored estimates backwards in time by taking $t = T, T-1, \dots, 2$. The initialization is evidently with the filtered estimate.

2.4 Spectral estimation

Once the time-varying coefficients of the ARMA(p,q) model (1) are solved the time-varying power spectral density (PSD) estimation can be obtained in the terms of the estimated coefficients

$$P_t(\omega) = \sigma_e^2(t) \frac{|1 + \sum_{k=1}^{q} b_k(t) e^{-i\omega k}|^2}{|1 + \sum_{j=1}^{p} a_j(t) e^{-i\omega j}|^2} \quad (16)$$

where $\sigma_e^2(t)$ is the prediction error variance. After the adaptive algorithm, used to estimate the time-varying ARMA parameters, converges power spectrum can be calculated for each time instant.

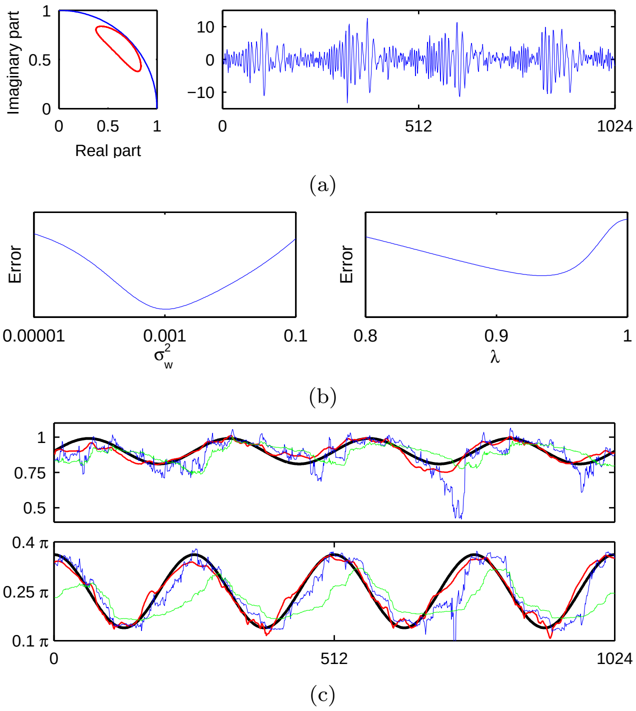

Fig. 1. AR(2) process estimation with RLS and Kalman smoother algorithms. The root evolution and the realization are presented in block (a). Both algorithms were optimized (b). Optimal value for the state noise covariance coefficient of the Kalman smoother was $\sigma_w^2 = 0.001$ and the forgetting factor of RLS was $\lambda = 0.935$. The estimates of the modulus and phase angle of the root are shown in block (c). The true values (black), Kalman smoother estimates (red) and optimal RLS estimates (blue). The smoother RLS estimates (green) were calculated by using $\lambda = 0.98$.

3 RESULTS

In order to evaluate the performance of the Kalman smoother algorithm we conduct two simulations, where Kalman smoother is compared to the popular forgetting factor RLS algorithm. Finally the Kalman smoother is applied to time-varying spectrum estimation of real EEG and for alpha wave detection.

3.1 Simulations

In the first simulation a time-varying signal was generated as an AR(2) process. The root evolution and a typical realization are presented in Fig. 1 (a). The modulus and phase angle of the root were estimated with Kalman smoother and RLS algorithms. Parameters controlling the adaptation were optimized in both algorithms to obtain the minimum error in AR coefficient estimation. The estimation errors as a function of adaptation parameters for both algorithms are presented in Fig. 1 (b). The estimates are shown in Fig. 1 (c).

RLS estimates with the optimal value for the forgetting factor have only a small tracking lag but the estimates are far more unstable compared to the Kalman smoother estimates. By increasing λ RLS estimates become more stable but the tracking lag increases at

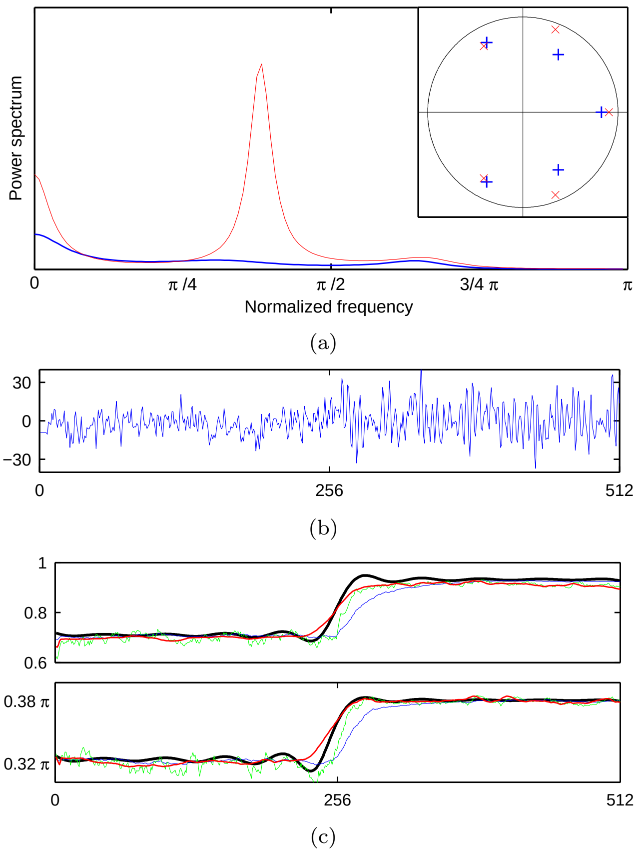

Fig. 2. A realistic simulation of EEG transition as an AR(5) process. (a) The roots and the corresponding spectra before (blue) and after (red) the transition. (b) A typical realization of the process. Averaged estimates over 100 realizations of the modulus and phase angle of the root corresponding to alpha activity are presented in (c), where true values (black), Kalman smoother estimates (red) and RLS estimates (blue/green) are shown. The state noise covariance coefficient of the Kalman smoother was $\sigma_w^2 = 8 \cdot 10^{-5}$ and the forgetting factors of RLS were $\lambda_1 = 0.98$ (blue) and $\lambda_2 = 0.9$ (green).

the same time. This simulation shows clearly the advantages of the Kalman smoother compared to the RLS algorithm. However not much can be said about the performance of the Kalman smoother in tracking of nonstationary EEG based on this simple simulation. Hence we aim to a more realistic simulation of EEG.

In many cases we are interested in tracking of narrow band characteristics of the EEG signal. One such case is the event related desynchronization/synchronization (ERD/ERS) of alpha waves. The occipital EEG recorded while patient having eyes closed shows high intensity in the alpha band (7-13 Hz). With the opening of the eyes this intensity decreases or even vanishes. It can be assumed that EEG exhibits a transition from a stationary state to another. Such a transition was here simulated as an AR(5) process. The roots of the system for both stationary states (obtained from real EEG measurements) and the corresponding power spectrums are presented in Fig. 2 (a).

In order to make the simulation more realistic abrupt transitions of AR coefficients were smoothed as described in [8]. A typical realization of the simulated

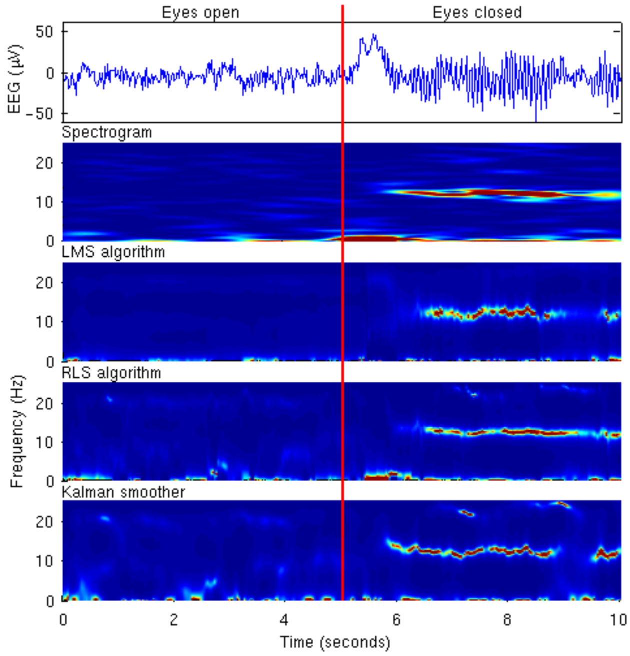

Fig. 3. Time-varying spectral analysis of ERD/ERS test of alpha waves of EEG. The measured EEG from channel O2 is shown on the topmost axis. The time window used in the spectrogram was 2 seconds. The step size of LMS algorithm was $\mu = 0.0002$ while the forgetting factor of RLS was $\lambda = 0.95$. The state noise covariance coefficient of the Kalman filter was $\sigma_w^2 = 0.0003$.

AR(5) process is presented in Fig. 2 (b). Results of tracking the alpha band characteristics are presented in Fig. 2 (c), where averaged estimates over 100 realizations of the phase angle and the magnitude of the root corresponding to alpha activity are presented. In order to obtain as smooth estimates with RLS as is obtained with Kalman smoother the forgetting factor λ must be quite small. However this leads to substantial tracking lag. With larger values of λ the tracking lag can be attenuated, but estimates become now more unstable.

3.2 ERD/ERS of alpha waves of EEG

The eyes open/closed test is a typical application of testing the desynchronization/synchronization of alpha waves of EEG. One such transition from desynchronized state to synchronized state is presented in Fig. 3. The performance of the Kalman smoother in tracking of alpha band characteristics is compared to most commonly used adaptive algorithms RLS and LMS and also to the traditional spectrogram method. An ARMA(6,2) model was used in all adaptive algorithms. The length of the time-window used in spectrogram was 2 seconds, which is long enough when considering the frequencies of the alpha band (7–13 Hz). The step size of the LMS algorithm was $\mu = 0.0002$ and the forgetting factor of RLS was chosen to be $\lambda = 0.95$ resulting in quite stable estimates and still rather fast adaptivity. The state noise covariance coefficient of the Kalman smoother was $\sigma_w^2 = 0.0003$.

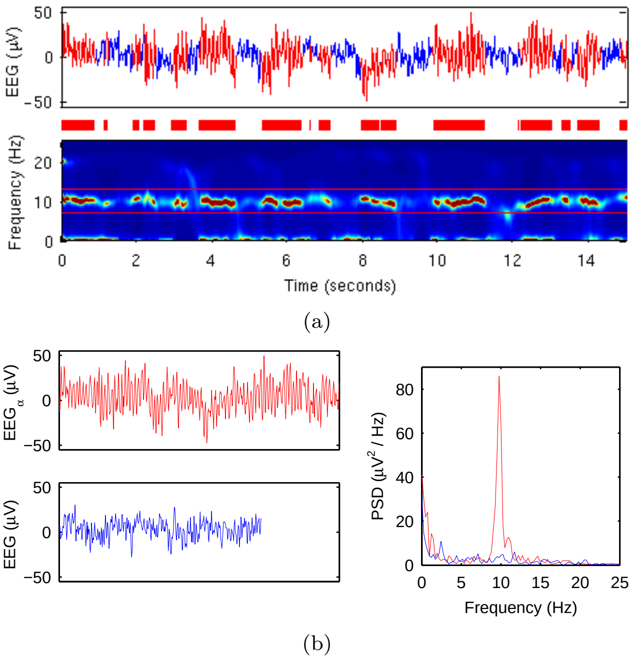

Fig. 4. Kalman smoother applied to alpha rhythm detection. (a) An EEG sample of 15 seconds from channel O2 measured from subject having eyes closed and the corresponding time-varying PSD. Detection is based on thresholding the power integral over the alpha band (7–13 Hz) with a threshold of 10 $\mu V^2/Hz$. Block (b) presents PSD estimates (calculated with traditional FFT based method) for the signals obtained by concatenating the EEG epochs where alpha activity was detected (red) or not detected (blue).

The tracking speed of the Kalman smoother seems to be fastest and an interesting gap in alpha rhythm is observed after 9 seconds. The contents of this kind of gaps is considered more closely in Fig. 4.

3.3 Detection of alpha rhythm of EEG

The aim of automatic EEG analysis is often the detection of certain waveforms. Hence the performance of the Kalman smoother on detection of alpha waves of EEG is considered here. Fig. 4 (a) presents a time-varying spectrum for an EEG sample of 15 seconds measured from healthy subject having eyes closed. Alpha wave detection was obtained by thresholding the power integral over the alpha band (7–13 Hz). The threshold was set to 10 $\mu V^2/Hz$. The performance of the alpha detector was verified by concatenating the EEG epochs where alpha waves were detected and those were no detection was made. The PSD estimates, calculated with a traditional FFT based periodogram method, for these concatenated signals are presented in Fig. 4 (c) verifying the absence of alpha rhythm in the lower concatenated signal.

4 DISCUSSION

The Kalman smoother algorithm was applied to tracking of nonstationary EEG. The performance of Kalman smoother in tracking of alpha band characteristics seemed to be most reliable compared to LMS and RLS algorithms. Kalman smoother was also applied to the detection of alpha waves of EEG with success. Also two simulations were conducted showing clearly the main advantages (smooth estimates without tracking lag) of Kalman smoother compared to other adaptive algorithms. The implementation and usability of the Kalman smoother approach are straightforward. The adaptation rate is adjusted simply by setting the state covariance coefficient σ_w^2.

One problem in modelling the data with adaptive algorithms is the selection of the model order. For time-invariant systems there exist various criteria for selecting the model order [9]. All these criteria are based on the compromise between model fit and model complexity. Also in the time-varying case there exist some criteria for selecting the model order. For example in [10] the use of Akaike's information criterion (AIC) was justified in the time-varying case under certain conditions. However in the case of tracking alpha rhythm of EEG the ARMA model of order $p = 6$ and $q = 2$ seems to be suitable. The same model order was also used in [11], [12].

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