TRACKING OF NONSTATIONARY EEG WITH THE ROOTS OF ARMA MODELS

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Abstract— **The tracking of nonstationary EEG with time-varying ARMA models is discussed. A method for detecting spindles in rat EEG is presented. The method is based on tracking of a single system pole of the ARMA model.** *Keywords:* **EEG, tracking, nonstationary, model.**

I. Introduction

The end to automatic analysis of EEG is often detection of certain waveforms or segmentation of the EEG into pseudostationary epochs and the subsequent classification of these. The parameters of autoregressive (AR) and autoregressive moving average (ARMA) models have been found to exhibit reasonably good discrimination efficiency in many cases [1]. However, there are situations where different classes would necessitate different orders of the model. The estimation of model orders for each segment would be computationally unfeasible and in practice we have to use orders that are sufficient for each class. This means that the variances of the parameter estimates will be large for the classes for which the orders are too high and the discrimination efficiency decreases. However, the variances of the roots of the model characteristic polynomials do not all behave like this. Some roots can retain small variances while the increase in the parameter variances affects other roots more than these.

The use of model roots has been proposed earlier for the classification of stationary epochs of EEG [2]. It has also been observed that epileptic seizures can be predicted by the movement of some roots in the complex plane [3].

We extend these results to time-varying EEG by using an adaptive predictor to estimate the model parameters from which the roots corresponding to the AR part (poles) are calculated. As an example we use this method to segment and classify the electrocorticogram of a drowsy rat.

II. Tracking of parameters

Time-varying ARMA(p,q) models for the process x_t can be written as

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + \sum_{\ell=1}^{q} b_\ell(t) e_{t-\ell} + e_t \ , \qquad (1)$$

where e_t is the prediction error process and the parameters $a_k(t)$ and $b_\ell(t)$ are estimated with an adaptive predictor. There are several algorithms that can be used as predictors. The most common ones are the LMS, RLS and the Kalman filters. We use here the recursive least squares (RLS) algorithm [4]. See [5] for discussion on the tracking of EEG with the Kalman filter. The forgetting-factor RLS algorithm takes the form

$$\begin{aligned}
X_t &= [x_{t-1}, \ldots, x_{t-p}, e_{t-1}, \ldots, e_{t-q}]' \\
e_t &= x_t - \theta'_{t-1} X_t \\
K_t &= P_{t-1} \frac{X_t}{\lambda + X'_t P_{t-1} X_t} \\
\theta_t &= \theta_{t-1} + e_t K_t \\
P_t &= \lambda^{-1} (I - K_t X'_t) P_{t-1}
\end{aligned}$$

where

$$\theta_t = [\hat{a}_1(t), \ldots, \hat{a}_p(t), \hat{b}_1(t), \ldots, \hat{b}_q(t)]' \qquad (2)$$

and the transpose is denoted by prime. To maintain the tracking capability of the algorithm we must have $\lambda < 1$ but otherwise the trade-off between tracking speed and estimate variance is controlled via the forgetting factor λ.

The calculation of all the roots from $\hat{a}_k(t)$ for each time can be computationally too expensive if some standard method, such as the calculation of the eigenvalues of the associated companion matrix, is used.

However, the approximation by tracking of a root or several roots based on previous estimates can be performed with various methods [6], [7]. We track a single root of the polynomial with one iteration of the Newton's method at a time.

III. Detection of Rat EEG Spindles

Rat EEG spindles are burst-like waveforms occurring when rats are drowsy. Their frequency of occurrence correlates e.g. with the learning capability and the effect of certain drugs. A typical epoch of rat EEG waveform is presented in Fig. 1. The spectral characteristics of spindles are distinguishable from that of background and therefore the use of parametric models in e.g. spindle detection is evident [8]. The rat EEG can be described as toggling between the spindle and non-spindle states. Both states can be modeled with ARMA$(4,2)$ models.

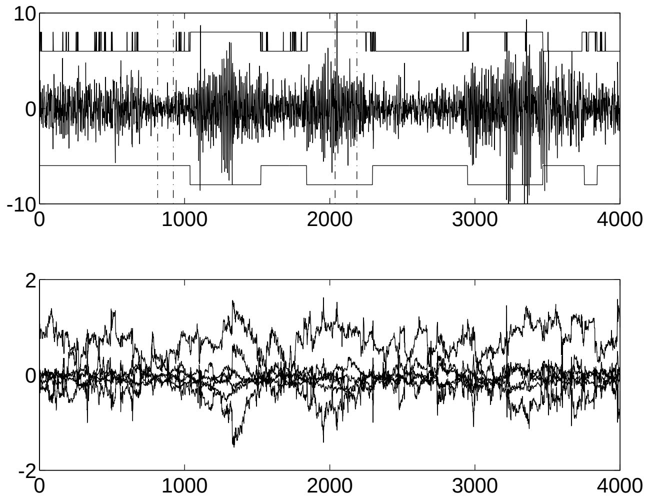

Fig. 1. a) The sample EEG signal and the result of the classification. The training intervals are denoted by vertical lines. b) The (time varying) ARMA -parameters of the model calculated with the RLS algorithm.

IV. The Method

The procedure of the spindle detection was as follows:

1. Select two segments from the data, one for each class. Use these as the learning sets for the classes.
2. Run these segments through RLS to obtain parameter estimates for the classes taking care to adjust the initial values.
3. Calculate the roots of interest and determine the classification (detection) boundary.
4. Run the whole data through RLS and a root tracker, classify and run further through an optional post-processor (e.g. a median filter).

V. Results

An example of rat EEG is shown in Fig. 1a. The training intervals are denoted by vertical lines in this figure. The time-varying $\mathrm{ARMA}(4,2)$ model was calculated with forgetting factor $\lambda = 0.97$. The poles of the model were calculated and one pole is shown in Fig. 2. Based on this a discriminant function dividing the complex plane into two half-planes was selected and is shown in Fig. 2. The whole data was modeled with with RLS and one system pole was recursively calculated using one step of Newton's method. The time evolution of the AR parameters for a sample of the rat electrocorticogram is shown in Fig. 1b. The preliminary classification was the based on the discriminant function and the location of the pole. The result of the classification is shown in Fig. 1a as a stair function above the EEG signal. The post-processed (a 80-point median filter) output of the classifier is seen inverted below the EEG signal.

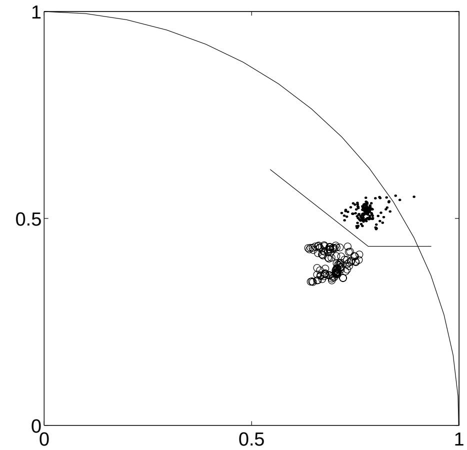

Fig. 2. The pole distribution of the model of Fig 1b on the training intervals. Only one pole is shown.

VI. Conclusions

The two states of the rat electrocorticogram can be efficiently discriminated by a pole of an $\mathrm{ARMA}(4,2)$ model. The tracking of the model parameters can be done efficiently with the forgetting factor RLS algorithm and the root of interest can be recursively approximated with the Newton's method. The computational burden is small enough to allow for a real time implementation with a general purpose personal computer. The results show that the method can be used to estimate the statistics of spindle occurrence, which is relevant to the problem.

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