PCA based Bayesian estimation of single trial evoked potentials

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Abstract: A method for the estimation of single trial evoked potentials is presented. The method is based on Bayesian Mean Squares estimation. The evoked potentials are estimated using the prior information obtained from the ensemble of the measurements. The partially noninformative prior covariance is formed using the eigenvectors of the covariance of the measurements. The performance of the method is tested using the visual evoked potential measurements.

INTRODUCTION

Over three decades it has been evident that amounts of useful information is lost when the evoked potentials (EP) are averaged [1]. This is caused by the fact that the components of the potentials are not allways in coherence between the repetitions of the stimulus. Variations in the latency and the amplitude of the components cause that the averaged evoked potential can differ very much from each single potential and does not correspond to any possible physical situation.

The multivariate analysis method called Principal Component Analysis (PCA) was first used for EP's in [2]. In PCA the stochastic sample is presented as a weighted sum of orthogonal basis vectors. The weights and the basis vectors are obtained from the eigendecomposition of the covariance matrix of the ensemble of the measurement vectors.

The fact that the basis functions in PCA are orthogonal has lead to misinterpretations that the basis functions could represent some independent physiological generators. This was disclaimed in [2] and [3].

Another approach to extract information beyond the average has been the filtering of the single trials of measurement. Time-varying Wiener filtering has been the approach that exhibits the most realistic assumptions about the signal [4], [5].

In this presentation we explain how the PCA and optimal filtering approaches can be combined and how this can seen as a way to form the Bayesian estimates for the underlying EP waveforms.

THEORETICAL BACKGROUND

Let the measured signal z be composed of EP signal s and background EEG signal v

$$z = s + v$$

In EP studies we usually make the assumption, that the EP is independent of the background EEG. Another assumption we make here is that the background EEG is a stationary zero mean process.

The definition of the correlation matrix R_s of the EP's is

$$R_s = E\left\{ss^T\right\}$$

so that it is an outer product of ensemble of rank one matrices. The realizations s span a vector space $\mathcal{S}$. In EP case $\mathcal{S}$ is spanned by those vectors which are possible outcomes of the experiment. Even if the outcome of the test is thought to be random, the dimension of $\mathcal{S}$ can be small or $\mathcal{S}$ can be approximated well with some low dimensional subspace of it. It is well known that the best approximation in the Mean Square sense is given by the eigendecomposition of the correlation or covariance matrix. This is the PCA approach.

We can thus express the EP's as linear combinations

$$s = K_{\mathcal{S}}\theta$$

where the columns of $K_{\mathcal{S}}$ are the eigenvectors of R_s. The task is now to estimate the parameters θ from the equation

$$z = K_{\mathcal{S}}\theta + v$$

It is well known that the Mean Square solution for this is

$$\hat{\theta}_{\rm MS} = (K_{\mathcal{S}}^T C_v^{-1} K_{\mathcal{S}} + C_\theta^{-1})^{-1}(K_{\mathcal{S}}^T C_v^{-1} z + C_\theta^{-1}\eta_\theta)$$

The best estimate for the EP is then

$$\hat{z}_{\rm MS} = K_{\mathcal{S}}\hat{\theta}_{\rm MS}$$

More usual is the Least Squares approach, where the covariance of the error is $C_v = I$ and the parameters are treated as nonrandom, $C_\theta^{-1} = 0$. In this case we have

$$z_{\rm LS} = K_{\mathcal{S}}(K_{\mathcal{S}}^T K_{\mathcal{S}})^{-1} K_{\mathcal{S}}^T z$$

If we select the observation model so that $s = \theta$, assume $\eta_\theta = 0$ (which is not realistic), $C_{v\theta} = 0$ and use the matrix inversion lemma once, we can write

$$\hat{\theta} = C_\theta(C_v + C_\theta)^{-1} z = C_\theta C_z^{-1} z = \hat{z}$$

which is just the equation of the Wiener filter used e.g. in [4], [5]. In these studies C_θ was parametrized and the time-varying filter was then solved.

Another approach is to use the observation model of the form

$$z = H\theta + v$$

with $H = I$, the identity matrix. The so-called regularized weighted Least Squares solution for this is

$$\hat{\theta}_\alpha = \arg\min_\theta \left\{ \|L_v(z - H\theta)\|^2 + \alpha \|L_\theta \theta\|^2 \right\}$$

where the operator L_θ defines the type of the regularization and $L_v^T L_v = C_v^{-1}$. If we select $L = (I - K_\mathcal{S} K_\mathcal{S}^T)$, the closeness of the solution $\hat{\theta}_\alpha$ to the subspace $\mathcal{S}$ can be controlled with the parameter α. In the limit we have $\hat{\theta}_\alpha \rightarrow \hat{\theta}_{\mathrm{MS}}$ as $\alpha \rightarrow \infty$. The solution to this minimization can be written in form

$$\hat{\theta}_\alpha = (H^T C_v^{-1} H + \alpha(I - K_\mathcal{S} K_\mathcal{S}^T))^{-1} H^T C_v^{-1} z$$

The best estimate for the EP is now

$$\hat{z}_\alpha = \hat{\theta}_\alpha$$

This approach can be seen to be equivalent to the Bayesian approach in Mean Squares estimation. The matrix $(I - K_\mathcal{S} K_\mathcal{S}^T)$ serves as the prior information about the EP's. It corresponds formally to the inverse of the covariance of the estimated parameters, in this case the EP's themselves because we use $H = I$. Because the matrix $(I - K_\mathcal{S} K_\mathcal{S}^T)$ is singular, the prior is partially noninformative. The prior variance is infinite in the subspace $\mathcal{S}$. This approach is not as restrictive as the PCA approach, where the solution is forced to be in $\mathcal{S}$.

RESULTS

We used the method for estimating the trials of the visual evoked potential measurement, so called P300 test. The first ten of the measured potentials are shown in figure 1. First we calculated the eigenvectors of the covariance matrix of the measurements. With these we approximated the matrix $K_\mathcal{S}$. We used the dimension of 4 for subspace $\mathcal{S}$. Another set of data was measured before the stimulus. We assume that these can be used for approximation of the statistical properties of the background EEG and calculated the matrix C_v. The regularized Least Squares solution $\hat{\theta}_\alpha$ was then calculated. The first ten estimates using $\alpha = 10$ are shown in figure 2.

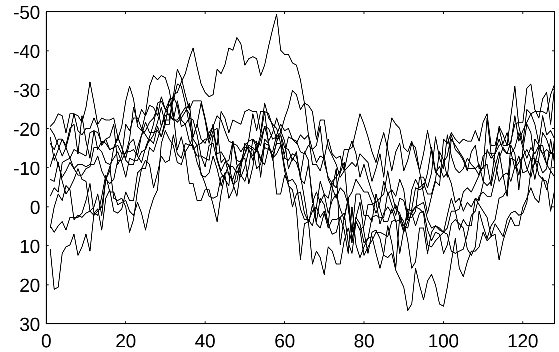

Fig. 1. Ten realizations of P300 measurements.

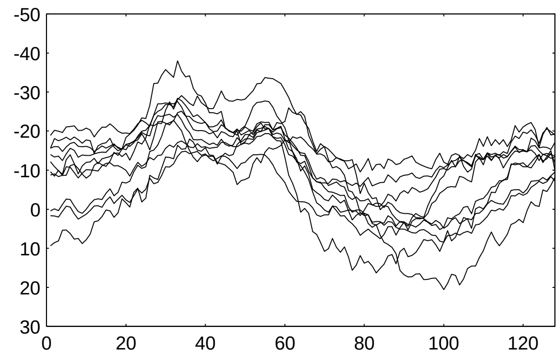

Fig. 2. Ten estimates of P300 measurements when four largest eigenvectors are used in regularization.

CONCLUSIONS

Principal component analysis can serve as tool in Bayesian estimation. With PCA it is possible to form a prior for single trial estimates in evoked potential analysis. With the approach presented here it is possible to include the prior information about the second order statistics that is available in the ensemble of measurements. Because the presentation of the method corresponds fully to the Bayesian approach in Mean Square estimation, it is possible to expand the results to include e.g. the time-varying case. This can be effective when the parameters of the components have trend.

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